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Unit 3

Operations with Fractions

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Mathematics 8

Learn  veryWare



Unit 3

Operations with Fractions

Mathematics 8
Unit 3: Operations with Fractions
Student Module Booklet
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- Learning Resources Centre, <http://www.lrc.education.gov.ab.ca>
- Tools4Teachers, <http://www.tools4teachers.ca>

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Unit 3: Operations with Fractions

Unit 3 Introduction



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Taking a trip across Canada—whether by train or by bus—is a spectacular experience. The ever-changing features of the landscape make the trip across this vast land so fascinating.

The differences in the physical features of Canada go beyond changes in the landscape. Climate, animal and plant life, and human activities change from one region to another.

When you look at the two-page spread on pages 194 and 195 of your textbook, you see a map of Canada. Canada is divided into regions called ecozones. Each ecozone has unique characteristics involving geography, climate, animal and plant life, and human activities.

Listen to the “Names of Ecozones” read out loud on the Math 8 Multimedia DVD. As you listen, follow the list of ecozones on page 195 of your textbook.

Besides their unique internal characteristics, ecozones in Canada differ vastly in the area they cover. Mathematical Operations with fractions and mixed numbers will help you describe and compare Canada’s various ecozones.

In Unit 3 you will study fractions and mixed numbers and discover how to multiply and divide with them. As you progress through this unit, you will gain the knowledge and skills needed to use fractions and mixed numbers in the description and comparison of ecozones.

In this unit you will multiply and divide with fractions and mixed numbers using models, diagrams, and symbols.

This unit will help you answer the following critical question: How can operations with fractions and mixed numbers be used to describe and compare ecozones?

The new concepts and skills with integers will be presented in six lessons.

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Unit 3: Operations With Fractions

Lesson 1: Using Models to Multiply Fractions and Whole Numbers

Lesson 2: Dividing a Fraction by a Whole Number

Lesson 3: Multiplying Proper Fractions

Lesson 4: Multiplying Improper Fractions and Mixed Numbers

Lesson 5: Dividing Fractions and Mixed Numbers

Lesson 6: Applying Fraction Operations

Operations with Fractions

In this unit you will be completing various types of assignments:

- interacting with other students through a discussion board
- adding samples of your work to your Math 8 folder
- completing sets of questions for each lesson
- solving a unit problem at the end of the unit
- possibly writing a unit test

Be sure to submit your assignments according to the directions that will be provided throughout this unit.

Strategies for Success

In order to support your success in this unit, follow these strategies.

Strategy 1

Make a foldable study tool according to the detailed instructions on page 196 of your textbook. Although this activity may not be graded for marks, you will benefit from this tool. Keep the points you record in mind as you develop and use this study tool.

- As you are working through the lessons, add formulas, examples, and vocabulary words.
- The foldable can serve as a quick reference guide and will help you save time when you are ready to study for a unit test or working on your unit problem.

Strategy 2

In this unit you will be referring to pages 194 to 241 of your textbook.

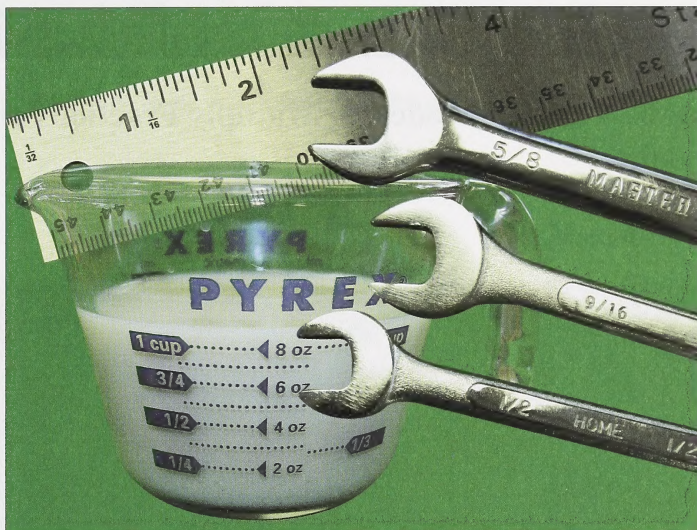
- Take time to flip through these textbook pages.
- Look at illustrations, margin features, and main titles to get a sense of where you will be going.

Unit 3 Problem

You will conclude this unit with a problem to solve. The unit problem is a comparison of Canada's ecozones using fractions and mixed numbers. You will complete the project according to the following textbook features:

- "Math Link" on page 203
- "Math Link" on page 209
- "Math Link" on page 215
- "Math Link" on page 221
- "Math Link" on page 229
- "Math Link" on page 235
- "Wrap It Up!" on page 239

These “Math Link” sections come at the end of textbook lessons. Each textbook lesson provides you with the skills you need to complete the questions in the lesson’s “Math Link” section. The “Math Link” on textbook page 197 will be used only to prepare you for the Unit Problem—that is, this “Math Link” section is not part of the Unit Problem itself.



The “Math Link” and “Wrap It Up!” textbook features describe ecozones ranging from the Arctic ecozone in the north to the Prairie ecozone in the south and from the Pacific Maritime in the west to the Taiga Shield in the east. You may want to read these textbook features now at one time so that you can picture the diversity of Canada’s geography and climate. Later, as you complete each textbook lesson, you may want to jot down some notes for the “Math Link” questions while the lesson ideas are still fresh in your mind. Your notes may help you as you complete the Unit Problem according to the directions you will find in the Unit Summary.

In Grade 7 you added and subtracted with fractions, so you are already familiar with fractions. You may have already discovered fractions along the sides of certain measuring cups, wrenches, and rulers. On these items, fractions are used to show volume, size, and length.

You should now be ready to learn more about fractions. But be careful with them. Fractions behave a little strangely—when you multiply a whole number by some fractions, you get a smaller value. When you divide a whole number by a fraction, you may get a larger value. You’ll find out more about fractions and mixed numbers in this unit. You won’t think their behaviour is so strange once you know more about them.

Unit 3: Operations with Fractions

Lesson 1: Using Models to Multiply Fractions and Whole Numbers

Get Focused



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The Prairie Ecozone runs up into the prairie provinces from the Canada-U.S. border. The northern boundary of this ecozone arcs from the southwest corner of Alberta to the southeast part of Manitoba. The main use of land in this ecozone is agriculture.

The photo gives you a view from the air of cropland in this region. During the growing season, the cropland takes on the pattern of a giant checkerboard. Of course, a real checkerboard is much more regular than the pattern shown in the photo.

Multiplying a fraction by a whole number can help you analyze coloured areas on ecozone maps and other surfaces.

In this lesson you will multiply fractions by whole numbers and solve problems with this operation.

This lesson will help you answer the following critical question: How can you model multiplication of a fraction by a whole number?

In this lesson you need the following, found on the Math 8 Multimedia DVD:

- “Fraction Strips”
- “Fraction Circles”
- “Pattern Blocks”

Note: Instead of using pattern-block templates, you may use the virtual manipulative “Pattern Blocks: Operations with Fractions.”



Assignments

Your assignment will consist of the following:

- posting to the discussion board
- adding to your Math 8 folder
- completing Unit 3: Lesson 1 Question Set

Explore

Multiplying a fraction and a whole number can give you the areas of ecozones. What does it mean to multiply a fraction by a whole number? How can a part of something be multiplied by a whole something? One way to create meaning for this operation is by creating a model using real objects. Then you can get an idea of how the numbers relate to one another by seeing how the model reacts.


Turn to “Explore the Math,” on pages 198 and 199 of your textbook. You’ll see great diagrams showing what it means to multiply a fraction by a whole number.

The “Explore the Math” is meant to be done with another person. Use your partner to help you understand the tasks and to share ideas on how to answer the questions. Your partner may be a classmate, friend, or someone on the discussion board.


Operations with Fractions

When you work through the “Explore the Math,” you will see that multiplying a **proper fraction** by a whole number can give you an answer that is more than one. This is modelled in question 1.a) The resulting answer can be expressed as an **improper fraction** or as a **mixed number**.

proper fraction: a fraction in which the numerator is smaller than the denominator

For example: $\frac{1}{2}$  Numerator is smaller than the denominator

improper fraction: a fraction in which the numerator is greater than the denominator

For example: $\frac{3}{2}$  Numerator is greater than the denominator

mixed number: a number written as a whole number and a proper fraction

For example: $1\frac{1}{2}$



Watch and Listen

You can see how improper fractions relate to mixed numbers in “Mixed Number Demo” on the Math 8 Multimedia DVD.



Self-Check

SC 1. Complete questions 1, 2, and 3 of “Explore the Math” on pages 198 and 199 of your textbook. You may use the multimedia piece “Pattern Blocks” instead of actual plastic pattern blocks.

Compare your answers in the Appendix.

My Guide

Q: With a yellow hexagon in a set of pattern blocks representing one whole, what fraction does a blue rhombus represent?

A: It takes three blue rhombus shapes to cover one yellow hexagon.



Therefore, each blue rhombus represents the fraction $\frac{1}{3}$.

Q: With a yellow hexagon in a set of pattern blocks representing one whole, what fraction does a green triangle represent?

A: It takes six green triangles to cover one yellow hexagon.



Therefore, each green triangle represents the fraction $\frac{1}{6}$.

Q: What addition do the pattern blocks in question 1.a) show?

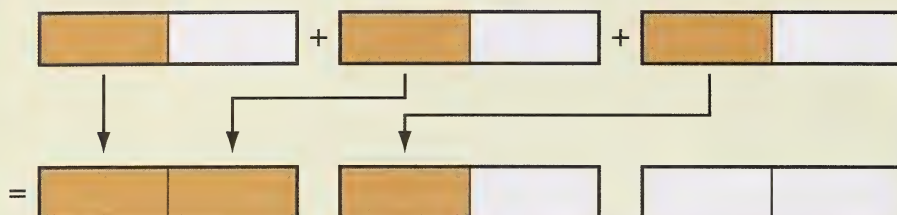
A: With each red trapezoid representing $\frac{1}{2}$, three of them represent the addition $\frac{1}{2} + \frac{1}{2} + \frac{1}{2}$.

Q: What multiplication is this addition the same as?

A: This addition is the repeated addition of $\frac{1}{2}$. Thinking of multiplication as repeated addition, this addition is the same as $3 \times \frac{1}{2}$.

Q: How would you use fraction blocks to represent the multiplication of $3 \times \frac{1}{2}$?

A:



You've now discovered ways to model the multiplication of a fraction and a whole number using manipulatives. Think back on your discoveries as you try the next questions.



Try This

TT 1. a. Using a method of your choice, model $4 \times \frac{1}{3}$. Use a diagram or a description to indicate how you modelled the multiplication; then write an equation to represent your model.

b. Give a good reason why you chose the method you used over other possible methods.



Discuss and Share

Post your responses to TT 1 on the discussion board. Then respond to at least two other postings.

Connect

Now it is time to build on what you've found out in the Explore.



Read

Read "Example 1: Multiply Using a Model" on page 199 of your textbook to find models for multiplication that make sense to you and that you can understand. You may prefer to use fraction strips, shapes, or the model shown on page 199.



Self-Check

SC 2. Use your preferred method to answer questions a) and b) from “Show You Know” at the top of page 200 of your textbook.

Compare your answers in the Appendix.



Read

There is another great way to model multiplication. Read “Example 2: Multiply Using a Diagram” on page 200 of your textbook to see how a number line can be used to model multiplication.



Self-Check

SC 3. Complete the “Show You Know” questions near the middle of page 200 of your textbook.

Compare your answers in the Appendix.



Read

Read “Example 3: Apply Multiplication With Fractions” on pages 200 and 201 of your textbook. Find out how multiplying the number of legs of a spider and a proper fraction give you the number of legs of an ant.

Maybe you enjoy baking. If so, then you know that with some recipes you end up baking much more food than you and a friend want to eat. Fractions help in adjusting recipes so that you make just the right number of squares or some other treat.



© Brooke Whatnall/Dreamstime

My Guide

Did you see the “Literacy Link” at the bottom of page 200 of your textbook? It tells you that you can often interpret *of* as multiplication. This can help you to solve word problems.

In the next “Show You Know” in your textbook, you will need to use $\frac{2}{3}$ *of* the number of scoops called for in the recipe.



Self-Check

SC 4. Find out how fractions are applied to recipes by doing the question from “Show You Know” near the middle of page 201 of your textbook. Use the My Guide button above if you need a hint to answer this question.

Compare your answers in the Appendix.



Read

Read “Key Ideas” on page 201 of your textbook as a short summary of this lesson. Make note of this for study purposes.



Try This

TT 2. Do questions 1, 2, and 3 from “Communicate the Ideas” on page 202 of your textbook.

My Guide

Q: If a yellow hexagon represents one whole, is there any shape in a set of pattern blocks that represents $\frac{1}{5}$?

A: No. The red trapezoids represent $\frac{1}{2}$, the blue rhombus represents $\frac{1}{3}$, and the green triangles represent $\frac{1}{6}$. There are no other shapes to represent $\frac{1}{5}$.



Place a copy of your answers in your Math 8 course folder.



Self-Check

SC 5. Do “Check Your Understanding” questions 4, 6, and 7 on pages 202 and 203 of your textbook.

SC 6. Do “Apply” questions 8, 9, 12, 13.a), and 14 on page 203 of your textbook.

Compare your answers in the Appendix.

Extra Practice

Still feel like a bit more practice is needed to fully understand this lesson? You may want to complete additional questions from “Practise” and “Apply” on pages 202 and 203 of your textbook for additional practice. You can then check your work using the answers given at the back of your textbook to see how well your understanding is coming along.



Assignment

Go to the Unit 3 Assignment Booklet, and complete the “Unit 3: Lesson 1 Question Set.” You may do your modelling with a drawing program or a simulation instead of using actual manipulatives. Taking screen shots is a fast way to record the steps you follow in your modelling.

Screen Shots

To take a screen shot of your virtual computer work, do the following:

- Make sure the work you want in your screen shot is clearly showing on your screen.
- Press the Print Screen button. It may appear as PrtSc on some keyboards. This key is usually located in the upper-left or upper-right corner of your keyboard. (If you want to capture just part of a screen shot, place your cursor on the window that you want to save. Then press Ctrl, Alt, and Print Screen.)
- Go to your MS Word document (or wherever you are completing your work).
- Paste the screen shot into your document. You can paste by pressing Ctrl V.

Going Beyond



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You may have watched a ball bouncing after it was dropped. It's fascinating how the ball hits the floor with ever-faster frequency until coming to a stop. Imagine trying to keep track of how far the ball travels while bouncing.

That would be difficult. But noting the fraction of the height the ball reaches after the first bounce will let you estimate the distance the ball travels over many bounces.

See how in the next question.

My Guide

Do you find modelling multiplication becomes tedious when large whole numbers are involved? Then you may find Nadine's method of multiplication helpful. Review question 3 of "Communicate the Ideas" on page 202 of your textbook.

Turn to page 203 of your textbook and do question 17 of "Extend."

Compare your answers in the Appendix.

Lesson Summary

In this lesson you multiplied fractions by whole numbers using a variety of models and solved problems with this operation.

You can model multiplication of a fraction by a whole number in a variety of ways. In this lesson you modelled multiplication with fraction strips, pattern blocks, fraction circles, number lines, rectangle diagrams, and counters and grids. You may also have developed some of your own ways of modelling this operation.

Unit 3: Operations with Fractions

Lesson 2: Dividing a Fraction by a Whole Number

Get Focused



Adapted from © Map Resources

Maybe you remember the first day that frost was predicted after the warm summer weather. If there were still flowers or tomatoes outside, you may have helped cover them so they could survive a few days longer.

The average number of days with frost varies with the ecozones across Canada. Iqaluit, in the Northern Arctic, has frost on most of the days of the year. Vancouver has the fewest days of frost in Canada.

Turn to page 204 of your *MathLinks 8* textbook. A student summarized the information given in the opening of the textbook lesson in a table:

| Comparing Climates Data | | |
|-------------------------|---|--|
| Location | Number of Days with Frost | Comments |
| Iqaluit | $\frac{3}{4}$ of the days of the year | Iqaluit has five times as many days with frost as Vancouver. |
| Vancouver | ?—to figure out once I know more about division of a fraction by a whole number | none |

Dividing a fraction by a whole number can be used to calculate the number of frost-free days in one ecozone based on the climate information of another. Dividing a fraction by a whole number can be used to solve other problems as well. Later in this lesson, filling in the missing data in the student's table will be possible.

In this lesson you will be learning to divide fractions by whole numbers and to solve problems with this operation. You know that dividing a whole number by another whole number sometimes gives you a fraction. But dividing a fraction by a whole number is something you may find a little harder to grasp at first. Modelling the operation will help you understand dividing a fraction by a whole number. Modelling with some of the familiar shapes or strategies you used in Lesson 1 will help you form an understanding.

This lesson will help you answer the following critical question: How can you model division of a proper fraction by a whole number?

In this lesson you need the following from the Math 8 Multimedia DVD:

- “Fraction Strips”
- “Pattern Blocks” or “Pattern Blocks: Operations with Fractions”



Assignments

Your assignment will consist of the following:

- posting to the discussion board
- adding to your Math 8 folder
- completing Unit 3: Lesson 2 Questions

Explore

If a fraction represents part of a whole number, what does it mean to divide a fraction by a whole number? To really get an understanding of what this operation means, you will complete “Explore the Math” on page 204 of your textbook. In doing this activity you may discover a way to model this puzzling operation.



Self-Check

SC 1. Complete “Explore the Math” questions 1 and 2 on page 204 of your textbook. Work with a partner, if possible, to help you discuss and share ideas. Your partner may be a classmate, friend, or family member.

Compare your answers in the Appendix.

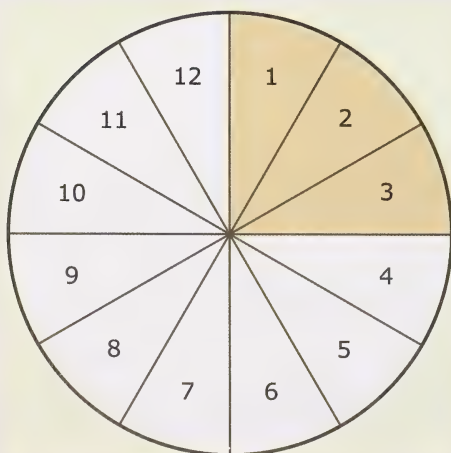
My Guide

Q: Think of the operation of division in the statement $12 \div 3 = 4$. What does the statement $12 \div 3 = 4$ mean?

A: The statement has more than one meaning. The statement can mean that when 12 objects are separated into 3 equal groups, each group will end up with 4 objects. The statement can also mean that when 12 objects are used to make groups each containing 3 objects, you'll end up with 4 groups.

Q: Suppose you were told that $\frac{1}{4} \div 3 = \frac{1}{12}$. That may still seem to be an unusual concept. How can it be interpreted to make its meaning clear?

A: This statement means that when $\frac{1}{4}$ of an object is separated into three equal parts, each part will be $\frac{1}{12}$ of the whole object. It may help you to imagine this: You have just finished Thanksgiving dinner, and the pumpkin pie is almost gone. There is only $\frac{1}{4}$ of the pie left (e.g., the orange area below). You and your two cousins still would like a piece of pie. If the remaining pie is divided equally, you will each get $\frac{1}{12}$ of the original pie.



Q: Can $\frac{1}{4} \div 3 = \frac{1}{12}$ also be interpreted the other way? A student started his attempt this way:
When $\frac{1}{4}$ of an object is organized into groups . . .

A: This interpretation would not work well. The fraction $\frac{1}{4}$ does not contain any whole number of groups where each group has three in it. In this lesson model division as separation into a whole number of equal parts.



Try This

TT 1. Share your models for division with others. Indicate a model that you prefer, and give a reason for your preference.



Discuss and Share

Post your responses to TT 1 on the discussion board. Then respond to at least two other postings. You may want to comment on how other models are the same or different from the models you created.

Connect

Now that you have done some exploration, it is important to build on the understandings you have developed so far.



Read

Maybe you prefer fraction strips to other manipulatives? Read “Example 1: Divide Using a Model” on page 205 of your textbook to see fractions strips used to model division.



Self-Check

SC 2. Try using the fraction strip method to complete the questions from “Show You Know” at the bottom of page 205 of your textbook.

Compare your answers in the Appendix.



Read

Perhaps a number line would appeal to the way you think about how the numbers interact in division. Read “Example 2: Divide Using Diagrams” on page 206 of your textbook to see how a number line can be used to model division.



Self-Check

SC 3. Try using the number line method by completing the questions from “Show You Know” near the middle of page 206 of your textbook.

Compare your answers in the Appendix.



Read

Find out just how much pasta sauce you have for six servings of pasta when you start with a partially empty jar of the sauce. Read “Example 3: Apply Division With Fractions” on pages 206 and 207 of your textbook. As you go over the example, you will also see how the number line is divided and further subdivided in order to give you the marks you need to place the brackets accurately.



Self-Check

It’s fairly easy—and enjoyable—to share a whole cake among four people. But if you only have half a cake to start with, you’ll end up with a smaller piece than when sharing a whole cake.

SC 4. Find out more about sharing half a cake. Do the question from “Show You Know” near the middle of page 207 of your textbook.

Compare your answers in the Appendix.



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**Read**

Read “Key Ideas” on page 207 of your textbook. It offers a short summary of this lesson. You might want to add the information from “Key Ideas” to a foldable like the one shown on page 196.

**Try This**

TT 2. Try your ability with the fraction strips and the number lines by completing questions 1 and 2 from “Communicate the Ideas” on page 208 of your textbook.



Place a copy of your answers in your Math 8 course folder.

**Self-Check**

SC 5. Complete “Practise” questions 4.a), 4.d), 5.a), and 5.d) on page 208 of your textbook to practise dividing fractions. Think about which model will you use to help you figure out the solutions.

Compare your answers in the Appendix.

SC 6. Complete “Apply” questions 6 and 8 on pages 208 and 209 of your textbook for some experience in problem solving.

Compare your answers in the Appendix.

Extra Practice

Turn to pages 208 and 209 in your textbook. For extra practice, you may complete additional questions from “Practise” and “Apply.” Then check your work using the answers given at the back of your textbook.



Assignment

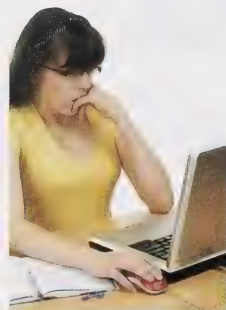
Contact your teacher to discuss any material, if you have any questions. When you feel you have a good understanding of the ideas presented in this lesson, go to the Unit 3 Assignment Booklet, and complete the “Unit 3: Lesson 2 Question Set.”

Going Beyond

In a division statement involving whole numbers, you can switch the divisor and the quotient. The resulting statement will still be true.

For example, you can switch the 3 and 4 in $12 \div 3 = 4$. The resulting statement, $12 \div 4 = 3$, is also true.

Can you also switch the divisor and the quotient in a statement showing a fraction divided by a whole number? Challenge yourself by trying to work through question 15 of “Extend” on page 209 of your textbook to see whether or not the divisor and the quotient can be switched.



© Andrzej Tokarski/Dreamstime

Compare your answers in the Appendix.

Lesson Summary

In this lesson you divided fractions by whole numbers and solved problems with this operation. You used models to make your own meaning of this kind of operation.

You can model division of a proper fraction by a whole number in a variety of ways. You used fraction strips, pattern blocks, fraction circles, number lines, and rectangle diagrams. You may also have developed some of your own ways of modelling this operation.

Unit 3: Operations with Fractions

Lesson 3: Multiplying Proper Fractions

Get Focused



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Canadian ecozones are too harsh for many of the world's organisms. For example, two-toed sloths are mammals adapted to tropical rainforests. In Canada, they can only be seen in zoos.

When two-toed sloths move, they move very slowly and they move only as much as they absolutely need to. If you think they are not energetic, you are right. In fact, two-toed sloths sleep most of the time.

You can find how the two-toed sloth's sleeping habits compare to other mammals by turning to page 210 in your textbook. Read just up to the heading "Explore the Math." You will be able to answer the questions posed in the reading when you have the skill and understanding to multiply proper fractions.

This lesson will help you answer the following critical questions: How can you model multiplication of a proper fraction by a proper fraction? Is there a rule that tells how to multiply such fractions?

Operations with Fractions

By the end of this lesson, you will be able to multiply two proper fractions and solve problems with this operation. Modelling this operation using the area of a sheet of paper and regions of a rectangle will help you make sense of this operation and help you to develop a rule to multiply fractions and to estimate the product.

In this lesson you need the following:

- 6 to 8 sheets of plain paper
- yellow and blue crayons or coloured pencils



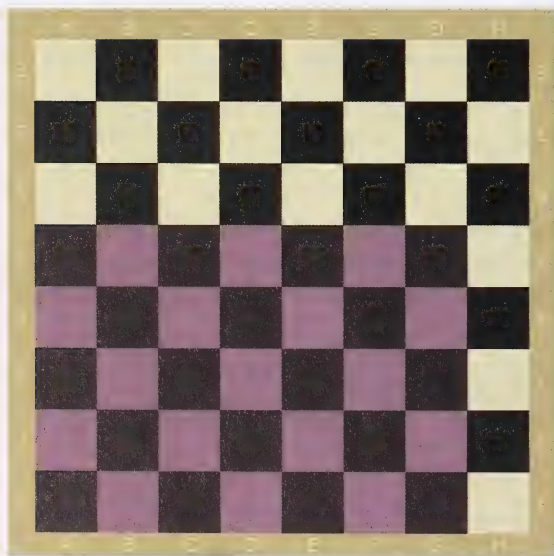
Assignments

Your assignment will consist of the following:

- adding to your Math 8 folder
- completing Unit 3: Lesson 3 Question Set

Explore

Some students were reviewing the operation of multiplication. They used rectangles on a checkerboard to model the multiplication of whole numbers. They agreed each square of the board—whether dark or light—should represent one unit of area. The side length of a square should represent one unit of length. Then the shaded rectangle they made over the board would represent the multiplication of the whole numbers 7 and 5. That is because the rectangle has dimensions of 7 units and 5 units. The product is 35—the number of squares covered by the rectangle.



Multiplying proper fractions involves factors that are less than 1. The checkerboard model above does not work because the factors are less than one unit and difficult to see because they would divide the board into hard-to-see pieces. For multiplication of factors less than 1, another model should be used. The next activity will help you develop a model for the multiplication of proper fractions.

The activity “Explore the Math” starts on page 210 of your textbook and finishes just before “Example 1: Multiply Using Paper Folding” on page 211. In doing this activity, you will develop and use a model to multiply two proper fractions. The materials you will need are the paper and pencils listed above.

Working with a partner in this activity would be beneficial for you as it will allow you to discuss the steps you are learning in this modelling process. Ask your teacher about who an appropriate partner would be. Your partner should be someone you can talk through the activity with and not just someone to check your work.



Self-Check

SC 1. Now it's your turn to try the paper folding. Complete questions 1, 2, 3, 4, and 5 of “Explore the Math.” If you use “1-cm grid paper,” found on the Math 8 Multimedia DVD, for your paper folding, you'll find paper folding is easier.

Compare your answers in the Appendix.

My Guide

Q: If you know the length and width of a rectangle, how can you calculate its area?

A: You multiply the length and width of the rectangle.



Look at the folded piece of paper.

Q: The horizontal fold (shown in blue) divides the shorter side of the rectangle into how many parts? What fraction does each part represent?

A: The horizontal fold divides the shorter side into two parts. Each of these parts represents the fraction $\frac{1}{2}$.

Q: The vertical folds (shown in red) divide the longer side of the rectangle into how many parts? What fraction does each part represent?

A: The vertical folds divide the longer side into three parts. Each of these parts represents the fraction $\frac{1}{3}$.



Look at the folded piece of paper. One of the six rectangles formed by the folds is shaded more darkly than any other. This rectangle is formed by the the overlap of two lightly shaped strips--one representing $\frac{1}{2}$ the page and the other $\frac{1}{3}$ of the page.

Q: What proper fractions can be used to represent the dimension of the most darkly shaded rectangle on the piece of paper?

A: Its length is $\frac{1}{2}$ and its width is $\frac{1}{3}$.

Q: Think of the area of the whole sheet of paper as one unit. Then what is the area of the most darkly shaded rectangle?

A: The area of the most darkly shaded rectangle is $\frac{1}{6}$. That's because it is one of six congruent rectangles making up one unit of area.

Q: What multiplication statement can you make for the folded-paper model?

A: $\frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$



Try This

TT 1. Complete question 6 of “Explore the Math” on page 211. Express your answers as a slide show.



Discuss and Share

Make a short slide show using a web cam to demonstrate the steps of both methods. Add narration to your slides to explain the steps you are illustrating. Exchange slide shows for TT 1 with another student. Use comments you receive on your slide show to polish your production.



Place your slide show in your Math 8 folder.

Connect

Let's now build on what you've found out in the Explore.



Read

After you've folded and shaded a sheet of paper, what parts of the diagram represent the numerator and denominator of the product? To find out, read “Example 1: Multiply Using Paper Folding” on pages 211 of your textbook.



Self-Check

SC 2. Try using the paper-folding method by completing the “Show You Know” questions at the top of page 212 of your textbook.

Compare your answers in the Appendix.



Read

Folding paper so that the folds are regularly spaced takes some care—even when using grid paper. Fortunately, drawings may be enough for you to visualize the operation of multiplication. Read “Example 2: Multiply Using Diagrams” on page 212 of your textbook to see how diagrams can be used to model multiplication. It helps to know that line segments for folds do not have to be placed exactly.



Self-Check

SC 3. Make your own diagrams to complete the “Show You Know” questions at the bottom of page 212 of your textbook. You may use the multimedia simulation “Exploring Fractions—Explore It” on the Math 8 Multimedia DVD. You will need your LearnAlberta user name and password to complete the diagrams for the questions. Select “Tooltips” and “Instructions” in the simulation to help guide you.

Compare your answers in the Appendix.

My Guide

When using Exploring Fractions—Explore It for multiplication, do the following:

- Count the number of boxes that make up the 1-by-1 grid.
- See the chosen fraction(s) shaded on the fraction grid.
- See that the overlapped shading of two fractions represents the result of multiplication.
- Identify the number of boxes in the 1-by-1 square will be the denominator resulting from multiplication.

From: *Exploring Fractions – Explore It; Junior High Interactives* © 2003-2009 Alberta Education. (www.learnalberta.ca)



Read

You may decide that all you need is the rule for multiplication to calculate the product. But without a visual representation, you may not notice if your calculation is wrong. In the next example, you will see how to estimate the product. Using the estimation, you can then judge whether your calculated answer is reasonable.

Read “Example 3: Multiply Using a Rule” at the top of page 213 of your textbook.

My Guide

In “Example 3” you see the numerators multiplied together. Then the denominators are multiplied together. If you remove common factors before multiplying, the calculation is simplified. Look at this other way of finding the product.

$$\begin{aligned}\frac{8}{15} \times \frac{5}{6} &= \frac{4 \times 2}{5 \times 3} \times \frac{5}{3 \times 2} \\ &= \frac{4 \times \cancel{2}}{\cancel{5} \times 3} \times \frac{\cancel{5}}{3 \times \cancel{2}} \\ &= \frac{4}{3 \times 3} \\ &= \frac{4}{9}\end{aligned}$$

By removing common factors first, the product comes out in lowest terms.



Self-Check

SC 4. Do the “Show You Know” questions near the middle of page 213 of your textbook. Remember to ask yourself if the product sounds reasonable based on your estimate.

Compare your answers in the Appendix.



Read

“Key Ideas” on page 213 of your textbook provides a short summary of this lesson. Make notes about this summary for your future study purposes. You might want to add these notes to a foldable like the one shown on page 196.



Try This

TT 2. Try your ability with the multiplication of fractions by completing questions 1 and 2 from “Communicate the Ideas” on page 214 of your textbook.



Use the discussion board to explain your methods and ask for some additional input. You can use the input from others to revise your answer. Then save this revised answer in your Math 8 course folder for your teacher to take a look at.

Think you have all the ideas in place for multiplying fractions using models? Challenge yourself with the following Self-Check questions. If you are struggling with any of the concepts, make sure to contact your teacher for additional help.



Self-Check

SC 5. Complete “Practise” questions 3 and 5 on page 214 of your textbook.

SC 6. Complete “Apply” questions 7, 11, and 12 on pages 214 and 215 of your textbook. Remember to ask yourself which modelling method will help you to solve the problems.

Compare your answers in the Appendix.

Extra Practice

Feeling like you need some extra challenge? You can do this by trying to complete additional questions from the "Practise" and "Apply" sections on pages 214 and 215 of your textbook. Brief answers are provided on page 493 for you to check your work. A partner may be helpful while you complete these questions so that you can talk through your thinking.



Assignment

Go to the Unit 3 Assignment Booklet, and complete the “Unit 3: Lesson 3 Question Set.”

Going Beyond

Many puzzles are based on whole numbers. For example, what two numbers have a product of 24 and a sum of 14?

By trying a few number pairs, you will see that 12 and 2 will work.

Puzzles with fractions are also fun, even if you don't see them so often. The next question is a puzzle in fraction multiplication.



© Jovan Nikolic/shutterstock

Turn to page 209 of your textbook and do question 15 of “Extend.”

My Guide

Keep the rule for multiplication in mind when solving part a. But use the rule backwards. Find the missing numerator by searching for a particular number. Multiplying this particular number by the known numerator (i.e., of one of the factors) should give you the numerator of the product. Use the same approach to find the missing denominator.

Bamboozled by part b? You may find that changing the product $\frac{1}{3}$ to the equivalent fraction $\frac{21}{63}$ may help solve the puzzle.

Compare your answers with the Appendix.

Lesson Summary

In this lesson you learned how to model the multiplication of proper fractions in two ways. First, you used areas on a folded sheet of paper, and then you used diagrams of a rectangle for the modelling process. From these models, you discovered the rule that tells how to multiply proper fractions: Multiply the numerators of each fraction and the denominators of each fraction to find the product fraction. Using this rule is the second way to multiply proper fractions. Also in this lesson, estimation was used to judge whether or not the products you solved for were reasonable solutions.

Unit 3: Operations with Fractions

Lesson 4: Multiplying Improper Fractions and Mixed Numbers

Get Focused

The shield on the Alberta flag illustrates several of the ecozones found in the province. Below the sky are the snow-capped mountains of the Mountain Cordillera. The rolling green hills correspond with the Boreal Plane. The Prairies ecozone, and human activity in that zone, are represented by the grassland and the field of wheat.

The width of the Alberta flag is $1\frac{4}{7}$ times the height of the shield displayed on it. The length of the flag is $3\frac{1}{7}$ times the height of the shield. How would you determine the size of an Alberta flag that has a shield $1\frac{1}{4}$ m high? You may have



© ARTROOM/Getty Images

decided that you will have to multiply the measurements but are unsure how to go about it. So far in this unit, you have only learned to multiply proper fractions. How do you multiply the mixed fractions shown? That is what you will be learning in this lesson, using models to help you.

In this lesson you will discover and apply rules to multiply with mixed numbers.

This lesson will help you answer the following critical questions: How can you model multiplication of improper fractions and mixed numbers? What straightforward rule can you use to find their product?



Assignments

Your assignment will consist of the following:

- posting and responding to the discussion board
- adding to your Math 8 folder
- completing Unit 3: Lesson 4 Question Set

Explore

How can you find the measurements of the Alberta flag described in the Get Focused section? You will need to be able to multiply with mixed numbers. The “Explore the Math” activity starts on page 216 of your textbook and finishes just before “Example 1: Multiply Mixed Numbers Using a Model” on page 217. In doing this activity, you will develop and use an area model to multiply mixed numbers.

The “Literacy Link” items located in the margins of pages 216 and 217 of your textbook provide review material. These links review what a **mixed number** is and what it means for a fraction to be in the lowest terms as well as how **improper fractions** and mixed numbers can be converted from one to another.

mixed number: a number written as a whole number and a fraction

improper fraction: a fraction in which the numerator is greater than or equal to the denominator

In exploring the operation of multiplication, you may want to create improper fractions in the multimedia animation Mixed Numbers. You’ll see how your improper fraction is connected to its equivalent mixed number.

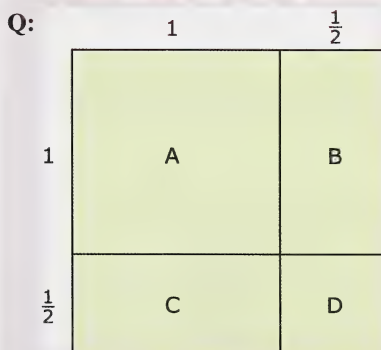


Self-Check

SC 1. Now it’s your turn to apply area models to the multiplication of mixed numbers. Complete questions 1, 2, and 3 of “Explore the Math” on pages 216 and 217.

Compare your answers in the Appendix.

My Guide



How else could you determine the total area of the large square?

A: Find the area of sections A, B, C, and D, and then add these areas.

Q: What is the connection between the dimensions (length and width) of each section and its area?

A: The product of the dimensions of each section gives the area of that section. Remember, for a rectangle: $\text{length} \times \text{width} = \text{area}$

Q: Which sections have 1 as a dimension? What are their areas?

A: Section A is 1 by 1; section B is 1 by $\frac{1}{2}$, section C is $\frac{1}{2}$ by 1. These areas all involve multiplication by 1. Section A has area 1; section B has area $\frac{1}{2}$; and section C has area $\frac{1}{2}$.

Q: What is the area of section D?

A: The area is found by finding calculating $\frac{1}{2} \times \frac{1}{2}$, which is a product of proper fractions. So, $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$.

Q: What is the total area of the large square?

A: The total area is the sum of the areas of the sections.

$$\text{area} = 1 + \frac{1}{2} + \frac{1}{2} + \frac{1}{4} = 2\frac{1}{4}$$

Bruce McAskill et al., *MathLinks 8 Teacher's Resource* (Toronto: McGraw-Hill Ryerson, 2008), 288. Reproduced by permission.



Try This

TT 1. Complete question 4 of “Explore the Math” on page 217 for the discussion board. You may present your steps in the form of plain text or, if you prefer, in a short slide show. Note that making a slide show for the first time may take lots of time.



Discuss and Share

Post your presentation to TT 1 on the discussion board. Then vote on whether you agree or disagree with at least four other versions of the rule from other students. Give a reason for why you agree or, if you disagree, explain what's wrong with the rule as expressed by another student. Remember to be courteous!

Connect

Let's now build on what you've found out in the Explore.



Read

What are the steps in modelling the multiplication of mixed numbers? To find out, read “Example 1: Multiply Mixed Numbers Using a Model” on pages 217 and 218 of your textbook.

In this example, note the following: In separating the dimensions of the large rectangle into a whole number and a proper fraction, you do not have to be exact. The lines just have to make four sections out of the large rectangle. These sections help you keep track of the parts that have to be added together for the product.



Self-Check

SC 2. Try using the area model by completing the “Show You Know” questions near the top of page 218 of your textbook. You may want to use the simulation “Exploring Fractions—Explore It” on the Math 8 Multimedia DVD to help you with the questions. You will need your LearnAlberta user name and password.

Compare your answers in the Appendix.



Read

Do you have to use diagrams each time you multiply mixed numbers, or can you follow a rule? Read “Example 2: Multiply Mixed Numbers Using a Rule” on page 218 of your textbook to find out.

When he was reviewing “Example 2,” Bret suggested that his group “look at the answer.” He pointed out the product. “Do you always end up with an answer that is larger than either factor when multiplying with mixed numbers? Or is the answer smaller than either factor or somewhere in between?” he asked the others. See if you can answer Bret’s question.

As you work through this example, you may choose to use the animation “Mixed Numbers” found on the Math 8 Multimedia DVD again. The animation will convert improper fractions of the solution to mixed numbers.



Self-Check

SC 3. Complete the “Show You Know” question at the top of page 219 of your textbook.

Compare your answers in the Appendix.



Read

Do you remember the important information from this lesson? Read “Key Ideas” on page 219 of your textbook for a short summary of this lesson. Make notes about this summary for your future study purposes. Now would be a good time to add these notes to a foldable like the one shown on page 196.



Try This

TT 2. Henri and Naomi need your help. Use your knowledge about mixed numbers to judge some students’ calculations involving mixed numbers. Complete questions 1, 2, and 3 from “Communicate the Ideas” on pages 219 and 220 of your textbook.



Place a copy of your answers in your Math 8 course folder.



Self-Check

SC 4. Complete “Practise” questions 4.a), 4.c), 5.d), 6.d), 7.c), 7.d), 8.b), and 8.c) on page 220 of your textbook.

Compare your answers in the Appendix.

SC 5. Now that you have completed some numerical problems, try some word problems multiplying mixed number and improper fractions. Complete “Apply” questions 10, 14, and 17 on pages 220 and 221 of your textbook.

Compare your answers in the Appendix.

Extra Practice

After checking your work in SC 4 and SC 5, were you able to accurately answer most of the questions? If you struggled on more than four questions, it would be a good idea to complete some more practice on this lesson. You can do this by turning to pages 220 and 221 in your textbook. For extra practice, you may complete additional questions from “Practise” and “Apply.” Then check your work using the answers given at the back of your textbook.

If you are still struggling, make sure to contact your teacher for additional support.



Assignment

Go to the Unit 3 Assignment Booklet, and complete the “Unit 3: Lesson 4 Question Set.”

Going Beyond

With a partner, you may want to play the “Fabulous Fractions” game on page 240 of your textbook. You have the background now to play parts 1 and 2 of the game.

In each of your turns, use a spinner to randomly select four digits. You use these digits to make two fractions that have as large a product as you can. If you can get a greater product than your opponent, you score a point.

For this game, you will have to make a spinner. It’s easy to make, and you can also use this spinner in parts 3 and 4. You may want to save these parts to play in a later lesson when you have more background information.

The “Template Spinner” on the Math 8 Multimedia DVD will provide a surface for your spinner. “The Multiplication Chart,” also on the Multimedia DVD, provides a place for you to record your results for part 2.a).

Have fun with the game!

You may also have fun looking for patterns. You can do this activity on your own.



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The cells of a honeycomb form a striking visual pattern. There may also be a pattern in a series of numbers. But to recognize a number pattern, you may have to do some investigation.

Turn to page 221 of your textbook, and use your skills with mixed number operations to solve “Extend” question 19.

Compare your answers in the Appendix.

Lesson Summary

In this lesson you multiplied improper fractions and mixed numbers. You found ways to model multiplication with mixed numbers. With the insight you gained through modelling, you came up with a rule for multiplying mixed numbers and improper fractions. Estimation was also used to judge whether products were reasonable.

You can model multiplication of mixed numbers by drawing a rectangle and dividing its dimensions according to the whole and fraction parts of the mixed-number factors. The areas of the sections combined represent the product.

You can multiply mixed numbers by converting them to improper fractions first. Then you can apply what you know about multiplying fractions to find the product. This idea led to a straightforward rule. This rule tells how to multiply mixed numbers and improper fractions:

- Convert any factor that is a mixed number to an improper fraction.
- Multiply the numerators and denominators of the factors.

In the next lesson you will divide with fractions and mixed numbers.

Unit 3: Operations with Fractions

Lesson 5: Dividing Fractions and Mixed Numbers

Get Focused



Adapted from © Map Resources.

Canada and Russia are the world's largest countries by area. Although these countries are on different continents, the Prairie ecozone is found in each of these countries. As is Canada, Russia is a major grain grower, and much of this grain is grown in its Prairie ecozone.

Turn to page 222 in your textbook *MathLinks 8*. One group of students summarized the information given in the opening of textbook lesson in a table.

AREA COMPARISON OF RUSSIA AND CANADA

| Country | Area Compared to Russia | Area Compared to Canada |
|---------|--|---|
| Canada | $\frac{3}{5} \times \text{Area of Russia}$ | n/a |
| Russia | n/a | $1\frac{2}{3} \times \text{Area of Canada}$ |

They did some research and summarized information relating the areas of Canada and the United States as well.

AREA COMPARISON OF THE UNITED STATES AND CANADA

| Country | Area Compared to Canada | Area Compared to the U.S. |
|---------|--|--|
| U.S. | $\frac{24}{25} \times \text{Area of Canada}$ | n/a |
| Canada | n/a | $1\frac{1}{24} \times \text{Area of U.S.}$ |

They noticed that in going from the first table to the second table, a larger fraction ($\frac{24}{25}$) leads to a smaller mixed number ($1\frac{1}{24}$). They wondered how the numbers in the cells of each comparison table related mathematically.

The mathematical relation that the group was puzzling over is a relation that will help you divide with fractions and mixed numbers.

By the end of the lesson, the mathematical relation between the two numbers in each of the comparison tables will become clear. You will even be introduced to a new term to describe the relation.

In this lesson you will discover and apply rules to divide with fractions and mixed numbers.

This lesson will help you answer the following critical questions: How can you model division with fractions and mixed numbers? What simple rule can you use to find the quotient?



Assignments

Your assignment will consist of the following:

- posting and responding to the discussion board
- adding to your Math 8 folder
- completing Unit 3: Lesson 5 Question Set

Explore

The “Explore the Math” activity is presented on pages 222 and 223 of your textbook. You will work through this activity as it is laid out in the text. When you work through “Explore the Math,” you will apply your knowledge of division with whole numbers. This knowledge will help you gain an understanding of division with fractions.

You may print out a “Fraction Division Table,” from the Math 8 Multimedia DVD which provides copies of the table in 5.a) and 6.a). The print out allows you to write entries directly in the tables cells.

Work with a partner, if possible. Your partner may be a classmate, friend, or family member. Discussing with others makes your learning so much easier.

When developing a rule for dividing fractions in “Explore the Math,” you may want to refresh your memory and review what the terms *dividend*, *divisor*, and *quotient* of a division statement mean by looking at the Math 8 Multimedia DVD.



Self-Check

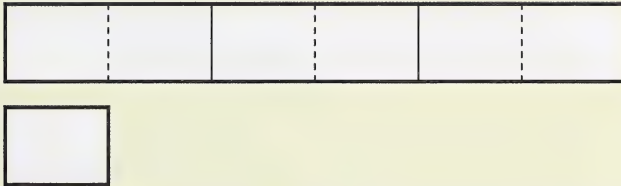
SC 1. Now you're the investigator. Use models and recognizing patterns to discover how to divide one fraction by another. Complete questions 1, 2, 3, 4, 5, and 6 of "Explore the Math."

Compare your answers in the Appendix.

My Guide

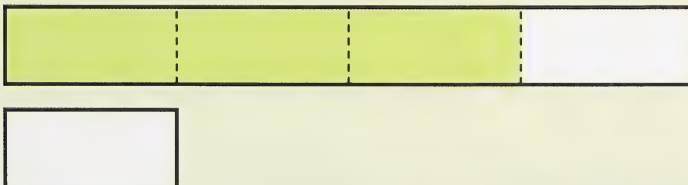
Q: How would I use manipulatives to help me with question 1.b?

A: You can cut out a long rectangular piece of paper to represent the dividend (3) and several shorter pieces of the same length of paper to represent the divisor ($\frac{1}{2}$). Then fit as many of the shorter pieces on the long piece as you can. Here are templates you can use for question 1.b.



Q: The dividend ($\frac{3}{4}$) is less than 1 in question 3.a. How would I use manipulatives to help me with this question?

A: This could be done with strips of paper cut out from these templates. See how many short pieces you can fit onto the shaded part of the long rectangle.





Try This

TT 1. A great thing about math is that there is almost always more than one way to get to an answer. Explain which method you prefer to use to divide fractions. Create a division problem, and use your preferred method to solve the problem.



Discuss and Share

Post your responses to TT 1 on the discussion board. Compare your solution to the solution completed by a person who used a different method. Think about advantages of your preferred method in solving the other person's problem. Post statements indicating advantages you see. Respond to at least one other person who comments about your posting.

Connect

Let's now reinforce what you and your partner discovered during the Explore.



Read

"Example 1: Divide Using Diagrams" on page 224 of your textbook shows how the common denominator is used to decide how fine to divide the rectangle. Work through this example. See how the bracket under the twelfths diagram makes four groups of thirds. Also note how the quotient compares to the original number being divided.



Self-Check

SC 2. Complete the questions from "Show You Know" in the middle of page 224 of your textbook for practise using this method.

Compare your answers in the Appendix.



Read

Find out how the **reciprocal** is used when the divisor is a fraction. Read “Example 2: Divide Using a Rule” on page 224 of your textbook. See how the quotient can be estimated.

reciprocal: the multiplier of a number that gives a product of 1

The reciprocal of a fraction is found by switching the numerator and the denominator of the fraction. For example, the reciprocal of $\frac{3}{7}$ is $\frac{7}{3}$ because $\frac{3}{7} \times \frac{7}{3} = 1$.



Discuss and Share

When you have completed “Example 2,” write the rule for dividing fractions in your own words or steps, and share this rule with others on the discussion board. Read other versions to see if you can improve your wording and place your final version in your foldable.



Self-Check

SC 3. Complete the “Show You Know” questions in the middle of page 225 of your textbook to familiarize yourself with completing this operation.

Compare your answers in the Appendix.

You can probably remember situations in which you applied the operation of division by a whole number to calculate an answer. For example, if you made \$120 for shoveling driveways, and you charge \$10 for a driveway, how many driveways did you shovel?



Read

“Example 3: Apply Division With Fractions” on page 225 shows how to apply division by a fraction. Not all problems in life will work out to whole numbers! As you read the example, think about these questions: Can you apply what you know about division with whole numbers when fractions are involved? What else do you have to remember to do when dividing with fractions?



Self-Check

SC 4. Complete the “Show You Know” question at the top of page 226 of your textbook.

Compare your answers in the Appendix.



Read

Read “Key Ideas” on page 226 of your textbook for a short summary of this lesson. Make notes about estimation and the use of common factors and reciprocals when dividing. Save these notes for your future study purposes. You might want to add these notes to the foldable you made like the one shown on page 196.



Try This

TT 2. Use your knowledge about division to complete questions 1, 2, and 3 from “Communicate the Ideas” on page 227 of your textbook. Apply the “Key Ideas” as you write out your answers for TT 2.



Place a copy of your answers in your Math 8 course folder. Your folder is a gold mine of solutions that help you when reviewing the concepts presented in the course.



Self-Check

SC 5. Do “Check Your Understanding” questions 5.a), 5.c), 6.a), 6.b), 7.a), 7.c), and 9 on page 227 of your textbook.

Compare your answers in the Appendix.

SC 6. Do “Apply” questions 11, 14, 16, and 18 on pages 228 and 229 of your textbook.

Compare your answers in the Appendix.

Extra Practice

Feeling like you need some more of a workout to really tone up your understanding? Then get together with a study partner, and turn to pages 220 and 221 in your textbook. Your partner could be someone from the discussion board. Complete a few additional questions from “Practise” and “Apply.” Then check your work using the answers given at the back of your textbook.



Assignment

Go to the Unit 3 Assignment Booklet, and complete the “Unit 3: Lesson 5 Question Set.”

Going Beyond

It seems that a person can learn loads from playing games—and have fun, as well. Find another math student to join you in a math board game called “Fabulous Fractions.” It’s described on page 240 of your textbook.

You may have played parts 1 and 2 in a previous lesson. You now have the background to play parts 3 and 4 of the game.

If you need to make a spinner, go to the “Template Spinner” on the Math 8 Multimedia DVD. You can record your results for part 2 using the “Multiplication Chart.” You can record your results for part 4 using the “Division Sheet.”

Lesson Summary

One of the critical questions that guided you in this lesson was the following: What simple rule can you use to find the quotient?

You found not one, but two simple rules that explain how to calculate the quotient when dividing with mixed numbers and fractions.

Rule based on use of common denominators:

- Change any mixed fractions into improper fractions.
- Express the dividend and the divisor as new fractions with a common denominator.
- Divide the numerators of the new fractions.

Rule based on multiplying by a reciprocal:

- Change any mixed fractions into improper fractions.
- Write the reciprocal of the divisor.
- Multiply the dividend by the reciprocal.

Over the whole lesson, you learned how to model division by a fraction with number strips. From this modelling and the patterns you observed, you found the two rules for dividing with mixed numbers. Also in this lesson, you solved problems with the operation of division and used estimation to judge whether or not solutions were reasonable.

In your next lesson you will apply the fraction operations you already know—addition, subtraction, multiplication, division—to solve word problems. The word problems become more fascinating to solve because you have to choose which of these operations to apply and in what order to apply them in a given problem.

Unit 3: Operations with Fractions

Lesson 6: Applying Fraction Operations

Get Focused

The ecozones in Canada are part of our natural heritage, which we want to preserve. Many Aboriginal people in Alberta are also worried about preserving their cultural heritage. Many young Aboriginal people now live in the cities. As a result, there is concern among some Aboriginal peoples that parts of their culture, such as language and traditional knowledge, may be lost.



© William Perry/iStockphoto

Fractions can help us understand the population trends affecting the preservation of cultural heritage. According to the last census, only about

$\frac{2}{9}$ of Alberta's 190 000 Aboriginal people live on reserves. Of those living off reserves, about $\frac{4}{5}$ live in urban areas.

Think about this question as mathematical problems to solve for a moment: About how many Aboriginal people live in urban areas? You will find that knowing which operations to apply and in what order is key to solving this question.

In this lesson you will solve problems involving operations with fractions. You will learn to identify which operation you need to solve the problem. When several operations are needed, you will learn to apply them in the proper order.

This lesson will help you answer the following critical questions:

- How do you solve a problem involving several operations with fractions?
- How do you choose the order in which to complete the operations?

You may need a calculator for this lesson. Also remember that the Toolkit may have tools to help you with your calculations.



Assignments

Your assignment will consist of the following:

- adding to your Math 8 folder
- completing Unit 3: Lesson 6 Question Set

Explore

How can you decide which operations to use when solving problems that have fractions? Such decisions are easier to make after sharing some exploration and discovery with others about using fractions in problem solving.

Work with others on the discussion board or in your class as you complete the following Self-Check questions. You may use a calculator to help you with calculations.



Self-Check

SC 1. Based on the information provided in the Get Focused, what is the population of Aboriginal people living on reserves?

Compare your answers in the Appendix.

My Guide

Q: From your reading, what expression represents those Aboriginal people living on the reserves?

A: The expression is “about $\frac{2}{9}$ of Alberta’s 190 000 Aboriginal people live on reserves.”

Q: What clue indicates that the number of Aboriginal people living on the reserves can be found by multiplying $\frac{2}{9}$ and 190 000?

A: The clue is the word *of*. In math, the word *of* often represents multiplication.

SC 2. What is the number of Aboriginal people that live in urban areas?

Compare your answers in the Appendix.

My Guide

Q: What sentence from the reading describes the number of Aboriginal people living in urban areas?

A: The sentence that describes the number of Aboriginal people living in urban areas is “Of those living off reserves, about $\frac{4}{5}$ live in urban areas.” This can be rephrased as “About $\frac{4}{5}$ of those living off reserves live in urban areas.”

Q: What do you need to do in order to use this sentence to calculate the number of Aboriginal people in urban areas?

A: You need to calculate the number of Aboriginal people living off reserves. You can do this by subtracting the number of Aboriginal people on reserves from the total number of Aboriginal people in Alberta.

SC 3. You can set up a mathematical expression made up of numbers, brackets, and mathematical-operation signs to represent the number of Aboriginal people in urban areas.

- a. Write a mathematical expression to represent the number of Aboriginal people in urban areas.
- b. Calculate the value of your mathematical expression.

Compare your answers in the Appendix.

My Guide

The following is an example of a mathematical expression made of brackets and other mathematical operation signs: $10 + \frac{2}{5} \times (12 - 2)$. To evaluate this expression, you first subtract inside the brackets—which equals 10—and then you multiply this value by $\frac{2}{5}$. This yields the value 4. Then you add this value of 4 to 10. This yields the value of 14, which is the value of the whole expression. In writing a mathematical expression for the population of Aboriginal people living in urban areas keep the order of operations in mind. Write the mathematical expression for this population so that you complete the mathematical operations in the same order as you did in calculating the answer to SC 2.

Suppose you had to only determine the population of Aboriginal people living in urban areas.

One method you used was to calculate in these steps:

- Calculate the population of Aboriginal people living on reserves.
- Calculate the population of Aboriginal people living off reserves.
- Calculate the population of Aboriginal people living in urban areas.

The other way was to calculate the population by writing an appropriate mathematical expression having brackets and mathematical operation signs and then evaluating that expression.



Try This

TT 1. Discuss with someone you know from your discussion board the two methods to solve the math problem about the populations of Aboriginal people. Agree to use instant messaging to persuade each other as to the better method. If you already prefer the same method of solving this math problem, discuss the pros and cons of both methods. As you communicate by online chat, react to each other's ideas. Do not just list ideas by yourself without listening to the other person.



Discuss and Share

Save your transcript from your instant messaging and place it in your Math 8 folder.

Connect

Let's now build on what you've found out in the Explore.



Read

What is the order of operations when evaluating a mathematical expression that has several operation signs? Read “Example 1: Use the Order of Operations” on page 231 of your textbook. Note that the mathematical operations are completed in the order indicated by the “Literacy Link” in the margin of the page.

My Guide

Need help in remembering the order of operations? Then think of BEDMAS. The letters stand for **B**rackets, **E**xponents, **D**ivision, **M**ultiplication, **A**ddition, and **S**ubtraction. This sequence may help you with order of operations:

- **B**rackets are done first.
- **E**xponents are next. (For example, the little raised 2 in $A = 5^2$.)
- **D**ivision and **M**ultiplication come next in the order you see them going from left to right.
- **A**ddition and **S**ubtraction together come last and are completed in the order you see them going from left to right.

You won't see many exponents in this lesson, but the E does make the letters easier to pronounce as a word. In the formula $A = \pi r^2$, the 2 is an exponent.



Self-Check

SC 4. Apply the order of operations in the “Show You Know” questions at the top of page 232 of your textbook.

Compare your answers in the Appendix.

**Read**

“Example 2: Apply Fraction Operations” on page 232 of your textbook shows why a sawmill worker may want to work extra hours in a week. See two methods of calculating the total amount earned when a worker works extra hours beyond the regular hours of work in a week.



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Look at the “Literacy Link” in the column of page 232 to see what it means to earn time-and-a-half. See where the mixed number $1\frac{1}{2}$ is used in the example solution. This mixed number is used as a factor so that the worker earns time-and-a-half for the extra hours worked.

**Self-Check**

SC 5. Complete the “Show You Know” question at the top of page 233 of your textbook.

Compare your answers in the Appendix.



Read

Do you remember the important information from this lesson? Read “Key Ideas” on page 233 of your textbook for a short summary of the lesson’s main ideas. Make notes about this summary for your future study purposes. You might want to add these notes to a foldable like the one shown on page 196.



Try This

TT 2. Apply the knowledge and skills of this lesson and help some students get concepts right. Complete questions 1, 2, and 3 from “Communicate the Ideas” on page 233 of your textbook.



Place a copy of your answers in your Math 8 course folder.



Self-Check

SC 6. Complete “Practise” question 4 on page 234 of your textbook.

Compare your answers in the Appendix.

SC 7. Solve “Apply” questions 6, 11, 12, and 13 on pages 234 and 235 of your textbook to practise creating your own numerical calculations based on word problems.

Compare your answers in the Appendix.

Extra Practice

Do you find the process tricky to understand? More practice can help you solidify the process in your mind. Turn to pages 234 and 235 in your textbook, and complete additional questions from “Check Your Understanding.” Then check your work using the answers given at the back of your textbook. If the concept is still giving you issues, be sure to contact your teacher to have a more in-depth discussion of the processes outlined in this lesson.



Assignment

Go to the Unit 3 Assignment Booklet and complete the “Unit 3: Lesson 6 Question Set.”

Going Beyond

Turn to page 235 of your textbook, and use your skills with fraction operations to solve “Extend” questions 16 and 17. These questions may look similar to what you have been solving, but they offer you more challenge. You almost have to solve these questions using a mathematical expression. You must write this expression by thinking more abstractly than the typical questions of this lesson. If you are still up to more challenge—go for it!

Compare your answers in the Appendix.

My Guide

Questions 16 and 17 may seem harder to you than they really are.

One way to show that this question is doable is to think of a question that is similar but simplified.

Look at question 16 again. Suppose that on an extraterrestrial piano there had 3 times as many white notes as black notes and a total of 88 notes. Then you know that the white notes make up $\frac{1}{4}$ of the notes. Divide 88 by 4 and you’d have the number of white notes. You ended up dividing by 4, which is 1 more than 3. The number 3 was the factor indicating how many more white notes there are than black notes.

In question 16 the number of white notes is $1\frac{4}{9}$ times the number of black notes. If you use a method that worked for the ET piano, you would find the answer by evaluating this expression:

$$88 \div \left(1 + 1\frac{4}{9}\right)$$

What about question 17? Think of the CDs $\left(1 + \frac{1}{2} + \frac{1}{4}\right)$ being in large racks. The brackets contain the fractions because the medium has only $\frac{1}{2}$ the capacity of the big one and the small one is only $\frac{1}{2}$ the capacity of the medium one. That means the small rack has $\frac{1}{4}$ of the capacity of the large one. It’s as if the CDs were in $1\frac{3}{4}$ large racks.

Now take a moment before getting back to the CDs in the large racks. Suppose that 2 cardboard boxes can hold 224 CDs. Then, to find the number of CDs that could be held in one cardboard box, you’d divide by 2.

So if $1\frac{3}{4}$ big racks hold 224 CDs, what do you divide 224 by to find the number of CDs in a big rack? You can likely answer this question by thinking back to the simplified problem involving CDs in cardboard boxes.

Lesson Summary

In this lesson you learned to solve problems involving multiplication, division, addition, and subtraction with fractions. You developed skill in deciding which of these mathematical operations had to be applied—and in which order—to solve a given problem.

One of the critical questions posed to guide your learning in this lesson was the following: How do you solve a problem involving several operations with fractions? You start by deciding on which mathematical operations you have to use. You base your decision on your interpretation of the problem. Then one method to solve the problem is by stages so that you only apply one of the mathematical operations at a time.

You can also solve the problem by evaluating an appropriate mathematical expression that includes all the mathematical operations related to the problem. Then you may recall the second critical question of the lesson: How do you choose the order in which to complete the operations? The order is as follows:

- **Brackets first**
- **Division and Multiplication next**
- **Addition and Subtraction last**

Division and Multiplication
are interchangeable.
So are Addition and Subtraction.
The operations on separate lines
are not interchangeable—
Multiplication can come after Division,
but not after Subtraction.

You've now completed the last lesson of the unit. In the unit summary, which comes next, you have an opportunity to apply the knowledge and skills of the whole unit.

Unit 3: Operations with Fractions

Unit 3 Summary

Getting Started



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A person who took a rail trip across Canada would go through numerous ecozones, each with its own geology, climate, and flora and fauna. Fractions and mixed numbers are useful in discussing the ecozones of Canada.

In the Unit Problem, you will use fraction and mixed-number operations to describe and compare ecozones in Canada. The Unit Problem is made up of questions in the following textbook features:

- “Math Link” on page 203
- “Math Link” on page 209
- “Math Link” on page 215
- “Math Link” on page 221
- “Math Link” on page 229
- “Math Link” on page 235
- “Wrap It Up!” on page 239

Operations with Fractions

These “Math Links” come at the end of textbook lessons. The textbook lessons provide you with the skills you need to complete the questions in each lesson's “Math Link.” The “Math Link” on page 197 is not listed. It will be used only to prepare you for the Unit Problem; this “Math Link” is not part of the Unit Problem itself.



Read

For a sense of how fractions and mixed numbers are used in talking about ecozones, read “Math Link” on page 197 of your textbook.

If you're not still sure about the pronunciation, listen to the “Names of Ecozones” read out loud on the Math 8 Multimedia DVD. As you listen, follow the list of ecozones on page 195 of your textbook.

You may print out a “Map of Canada” from the Math 8 Multimedia DVD that you can write notes and draw on. You can use this map as a reference, along with the labelled map on pages 194 and 195 of your textbook.

Much of this “Math Link” is based on the addition and subtraction of fractions and mixed numbers. You learned about these operations before starting this course.



Self-Check

SC 1. Complete questions 1 and 2 of the “Math Link” on page 197 of your textbook. If possible, work with a partner. You and your partner will help each other trigger previous learning about addition and subtraction of fractions and mixed numbers.

Compare your answers in the Appendix.

The Self-Check question you just completed helped you prepare for the Unit Project. Your project consists of responding to the questions that follow.

Project



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In travelling across Canada by rail or road, you would cross terrestrial ecosystems but may miss out on the marine ecosystems. So you would never see a whale leaping out of the surface of the water or other wonders of the ocean.



Try This

TT 1. The “Math Link” on page 203 draws your attention to two classes of ecozones in Canada—**marine** and **terrestrial**. Complete questions a) and b) to help you distinguish between the two classes. You could count the

zones directly off the visuals on textbook pages 194 and 195. However, try to use multiplication with a fraction to answer the questions. You can use the visuals to confirm your answer.

marine: of or relating to the sea or ocean

terrestrial: of or relating to the land as distinct from the sea, ocean, or air



Place a copy of your answers in your Math 8 course folder.



© Gary Hartz/Dreamstime

You may have visited the Montane Cordillera if you have travelled to Jasper. The Boreal Cordillera is farther north, covering the very northern part of British Columbia and much of southern Yukon.

My Guide

Q: Suppose you and a friend want to pool your money together to make a purchase. When you see how much money you have with you, you discover you both have the same amount and that, together, you have \$12. What mathematical operation would a person use to calculate how much each of you had? How much money did each of you have?

A: You would divide. You would divide the total you had together by 2.

$$12 \div 2 = 6$$

Each of you had \$6.



Try This

TT 2. Complete the “Math Link” question on page 209 of your textbook to learn more about the sizes of the Montane Cordillera and the Boreal Cordillera.

TT 3. Complete the “Math Link” question on page 215 of your textbook to find out the area covered by the Pacific Maritime ecozone.

My Guide

Think back to your solution of question 7 of “Apply” on page 214. For that question, you had $\frac{1}{4}$ of $\frac{1}{2}$ of the whole pie.

In TT 3 you are to find $\frac{1}{5}$ of $\frac{1}{10}$ of the area of Canada. You use the same mathematical operation.

TT 4. Complete the “Math Link” question on page 221 of your textbook to find out the area covered by Northern Arctic ecozone.



Place a copy of your answers in your Math 8 course folder.

Most of Canada’s grain is grown in the Prairie ecosystem. The precipitation in this ecosystem is critical to good crops. Calculate the precipitation in the dry grassland portion of the Prairie ecosystem in the next question.



Try This

TT 5. Complete the “Math Link” question on page 229 of your textbook.

My Guide

Q: Substituting the rounded value of the mixed number in the information will help you get started. So, if you were rounding, how could you read the final statement that says, “This amount of precipitation . . .”?

A: The amount of precipitation is 3 times the amount in the dry grasslands.

Q: With the rounded value in the statement, what would the answer be approximately?

A: $70 \div 3 = 23$

The average annual precipitation in these grasslands is approximately 23 cm. In your solution you should replace the 3 with the $2\frac{4}{5}$ and re-do the calculation. The approximation can help you determine if your calculated answer is correct since your calculated answer should be close in value to your rounded approximation.



Place a copy of your answers in your Math 8 course folder.

The species of mammals are not distributed evenly among the 20 ecozones of Canada. The next question will give you more insight into this distribution.



Try This

TT 6. Complete the “Math Link” question on page 235 of your textbook.

My Guide

You will need to apply two operations. You may find it easier to work out the number of mammal species in all of Canada first. Then subtract the number in the Taiga Shield.



Place a copy of your answers in your Math 8 course folder.

You have found facts about many ecozones in Canada by applying your skills with fractions and mixed numbers. As a concluding research activity, apply your skills to the Boreal Plains.

Turn to “Wrap It Up!” on page 239 of your textbook and read the information about the Boreal Plains. This description appears just before the questions.



Try This

TT 7. Do questions a) and b) of “Wrap It Up!” on page 239 of your textbook. Use feedback to make sure your three problems are well expressed and can be solved by others.



Place a copy of your three word problems and the solutions in your Math 8 course folder.

Unit Summary

In Unit 3 you studied fractions and mixed numbers. You discovered how to multiply and divide using models, diagrams, and symbols. You found that operations with fractions and mixed numbers allowed you to describe and compare ecozones.

Unit Review

The next Self-Check question will provide a selection of questions that will make you think back to each of the lessons of this unit.



Self-Check

SC 2. Turn to “Chapter 6 Review” on pages 236 and 237. Complete questions 1, 2, and 3; then answer at least two questions from each of the six lesson components. You may do additional questions for practice.

Compare your answers in the Appendix.

Are You Ready?

You’ve done a lot of work to reach this point in the unit. But are you ready to strut your stuff—that is, show your mastery of the new concepts and skills? Are you really ready to take the challenge of a unit test?

You may feel you are ready but you want to do a practice test before doing one for marks. Well, here’s your chance. Complete the following Self-Check question.



Self-Check

SC 3. Turn to pages 238 and 239 and complete “Practice Test.” Challenge yourself and treat this practice test as if it were a real unit test. See how well you do when you compare your answers. Review any sections you still might have questions about at this point.

Compare your answers in the Appendix.



Assignment

Check with your teacher about a unit test.

Congratulations! You’ve just completed another unit of study—way to go!

Appendix

Lesson 1

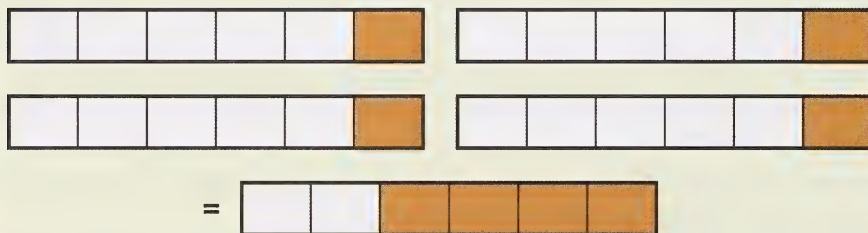
SC 1.

1. a. To the left of the equal sign, the three trapezoids model $\frac{1}{2}$ being added three times. To the right of the equal sign, the two stacked trapezoids model one whole and the other trapezoid models $\frac{1}{2}$.

b. To the left of the equal sign, the three trapezoids model $\frac{1}{2}$ being multiplied by 3. To the right of the equal sign, the two stacked trapezoids model one whole and the other trapezoid models $\frac{1}{2}$.

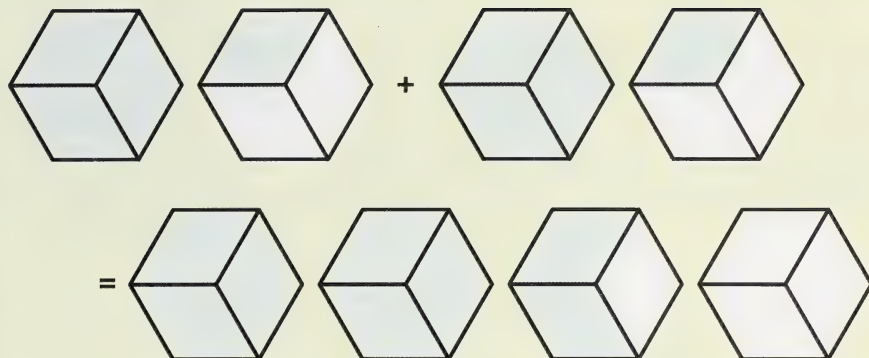
c. Examples of other manipulatives are fraction strips and fraction circles.

2. a. Answers will vary. Example:



b. $4 \times \frac{1}{6} = \frac{4}{6}$

3. a. Answers will vary. Example:

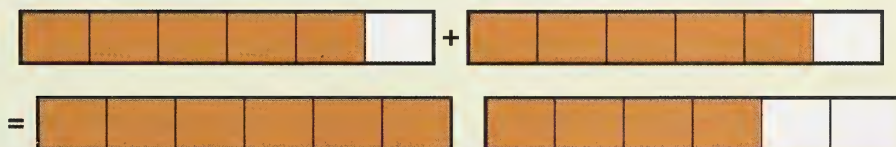


b. $2 \times \frac{4}{3} = \frac{8}{3}$

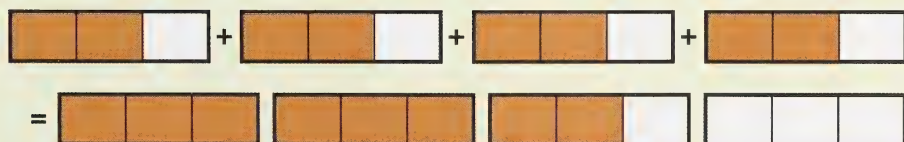
Bruce McAskill et al., *MathLinks 8 Teacher's Resource* (Toronto: McGraw-Hill Ryerson, 2008), 264. Reproduced by permission.

SC 2.

a.



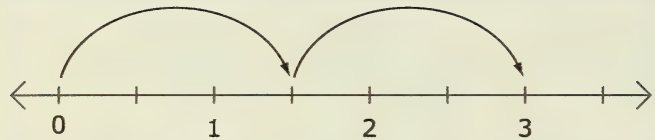
b.



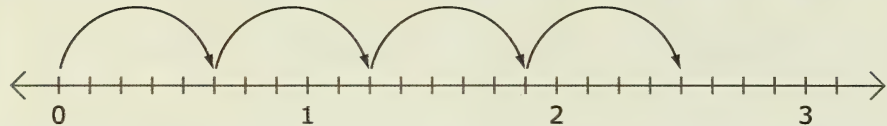
Bruce McAskill et al., *MathLinks 8 Teacher's Resource* (Toronto: McGraw-Hill Ryerson, 2008), 264. Reproduced by permission.

SC 3.

a.



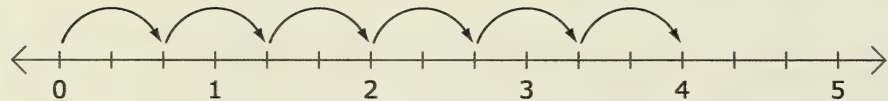
b.



Bruce McAskill et al., *MathLinks 8 Teacher's Resource* (Toronto: McGraw-Hill Ryerson, 2008), 264. Reproduced by permission.

SC 4.

To make $\frac{2}{3}$ of the recipe, all the quantities used in the recipe must be multiplied by $\frac{2}{3}$. One of the quantities of the recipe is 6 scoops of flour. Determine $6 \times \frac{2}{3}$. Model the multiplication as repeated addition on a number line.

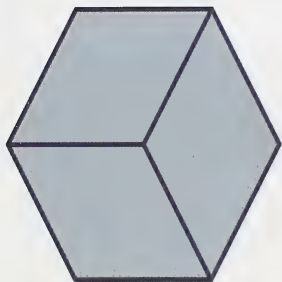


The result is 4. So $6 \times \frac{2}{3} = 4$.

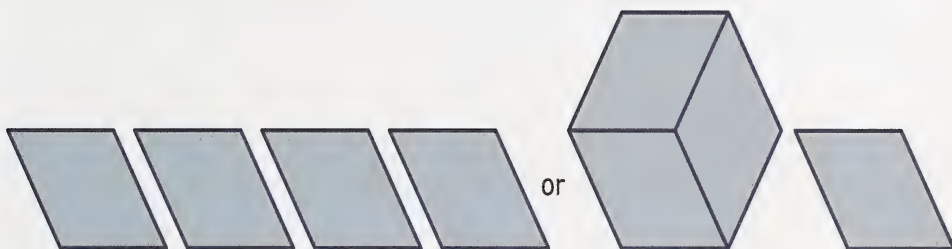
Jenelle will need 4 scoops of flour.

SC 5.

4. a.

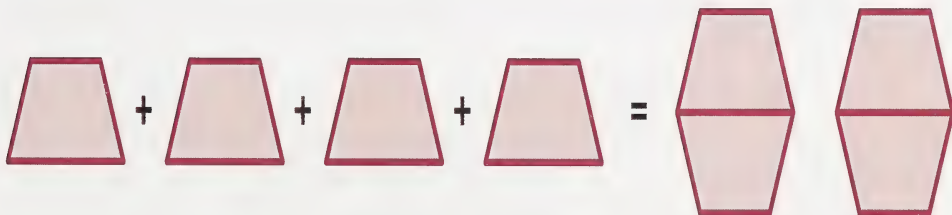


Since a rhombus represents $\frac{1}{3}$, the diagram represents $4 \times \frac{1}{3} = \frac{4}{3}$ or $1\frac{1}{3}$.



b. Since each fraction strip is divided into five parts and two are shaded, each diagram represents $\frac{2}{5}$. The series represents $3 \times \frac{2}{5} = \frac{6}{5}$ or $1\frac{1}{5}$.

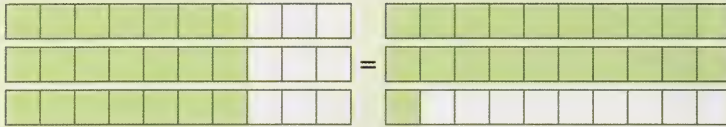
6. a. Each trapezoid represents $\frac{1}{2}$.



The diagram represents $4 \times \frac{1}{2} = \frac{4}{2} = 2$.

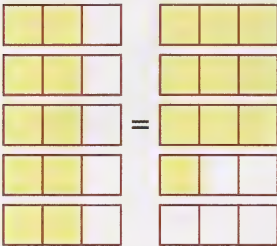
Operations with Fractions

b. Each rectangle is divided into three sections. Since seven of the ten sections are coloured, each rectangle represents $\frac{7}{10}$.



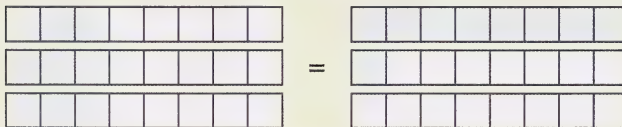
The diagram represents $3 \times \frac{7}{10} = \frac{21}{10}$ or $2\frac{1}{10}$.

c. Each rectangle is divided into three sections. Since two of the three sections are coloured, each rectangle represents $\frac{2}{3}$.



The diagram represents $5 \times \frac{2}{3} = \frac{10}{3}$ or $3\frac{1}{3}$.

d. Each rectangle is divided into eight sections. Since three of the eight sections are coloured, each rectangle represents $\frac{3}{8}$.



The diagram represents $3 \times \frac{3}{8} = \frac{9}{8}$ or $1\frac{1}{8}$.

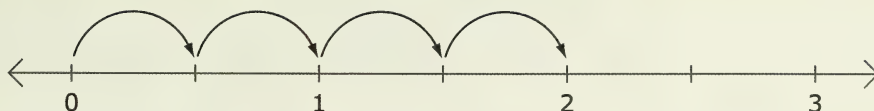
7. c. Each fraction strip is divided into five parts. Since six of the ten parts in two strips are coloured, each diagram represents $\frac{6}{5}$.



The diagram represents $2 \times \frac{6}{5} = \frac{12}{5}$.

SC 6.

8. One-half of four means $\frac{1}{2} \times 4$. This is the same as $4 \times \frac{1}{2}$. Model the multiplication as a repeated addition on a number line.

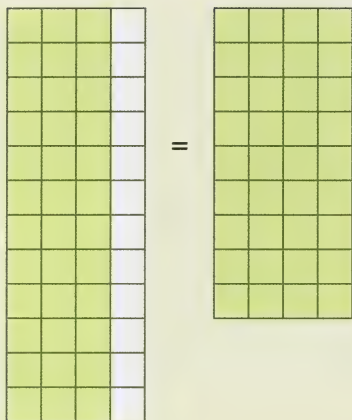


$$4 \times \frac{1}{2} = 2$$

The width of the flag is 2 m.

Operations with Fractions

9. To determine the number of people seated, multiply 12 by $\frac{3}{4}$. Use rectangles to model.



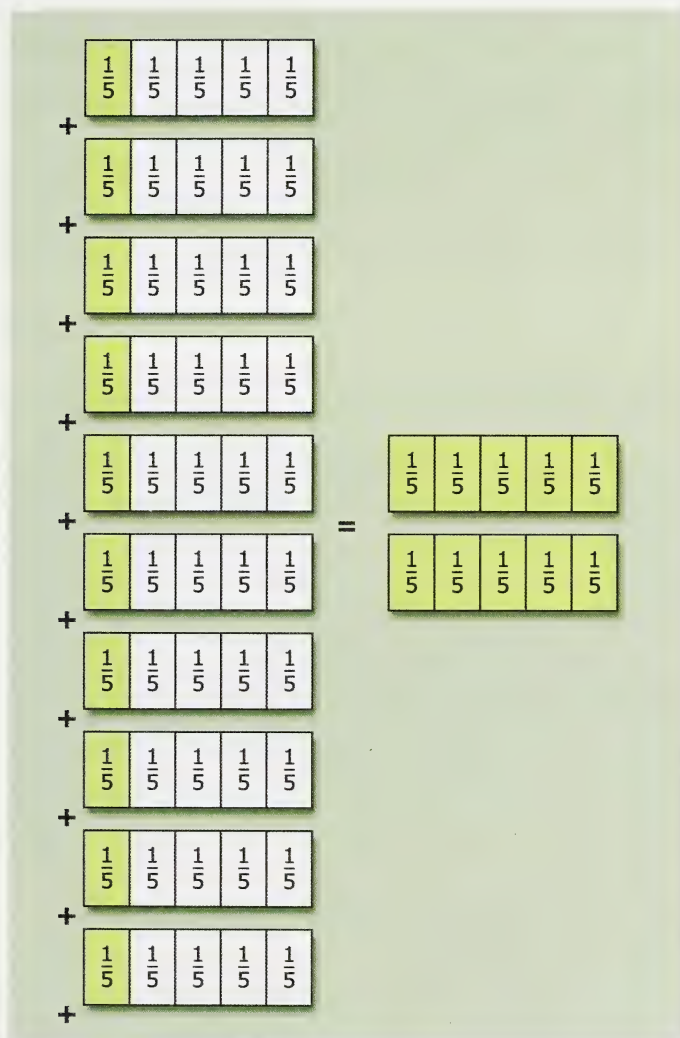
Each rectangle is divided into four sections. Three of the four sections are coloured, so each diagram represents $\frac{3}{4}$.

So, $12 \times \frac{3}{4} = 9$.

There are nine people seated in the minibus.

12. The area of Nunavut is $\frac{1}{5}$ of 10 million km^2 .

The equation of $\frac{1}{5} \times 10$ can be modelled this way:



$$\frac{1}{5} \times 10 = 2$$

The approximate area of Nunavut is 2 million km^2 .

13. a. The pattern is to divide to the previous product by 2.

Since $10 \div 2 = 5$, $\frac{1}{2} \times 10 = 5$.

Operations with Fractions

14. Answers may vary. The following is an example word problem:

Jane spends $\frac{1}{4}$ of her allowance on books. If her allowance is \$8, how much does she spend on books?

Answer to the word problem:

To find how much she spends on books, multiply $\frac{1}{4}$ by 8.

$$\frac{1}{4} \times 8 = \frac{8}{4} = 2$$

She spends \$2 on books.

Going Beyond

17. Distance for first drop: 81 cm

Distance for return bounce, $\frac{2}{3}$ of 81, is the same as $\frac{2}{3} \times 81 = 81 \times \frac{2}{3}$. Find the product.

$$81 \times \frac{2}{3} = \frac{162}{3} = 54 \rightarrow 54 \text{ cm}$$

Distance for the second drop: 54 cm

Distance for return bounce, $\frac{2}{3}$ of 54, is the same as $\frac{2}{3} \times 54 = 54 \times \frac{2}{3}$. Find the product.

$$54 \times \frac{2}{3} = \frac{108}{3} = 36 \rightarrow 36 \text{ cm}$$

Distance for third drop: 36 cm

Distance for return bounce, $\frac{2}{3}$ of 36, is the same as $\frac{2}{3} \times 36 = 36 \times \frac{2}{3}$. Find the product.

$$36 \times \frac{2}{3} = \frac{72}{3} = 24 \rightarrow 24 \text{ cm}$$

Distance for the fourth drop: 24 cm

Distance for return bounce, $\frac{2}{3}$ of 24, is the same as $\frac{2}{3} \times 24 = 24 \times \frac{2}{3}$. Find the product.

$$24 \times \frac{2}{3} = \frac{48}{3} = 16 \rightarrow 16 \text{ cm}$$

Distance for the fifth drop: 16 cm

Total distance travelled: $81 + 2 \times 54 + 2 \times 36 + 2 \times 24 + 2 \times 16 = 341 \rightarrow 341 \text{ cm}$.

The total distance travelled is 341 cm.

MathLinks 8: Solutions CD (Toronto: McGraw-Hill Ryerson, 2008), 6. Reproduced by permission.

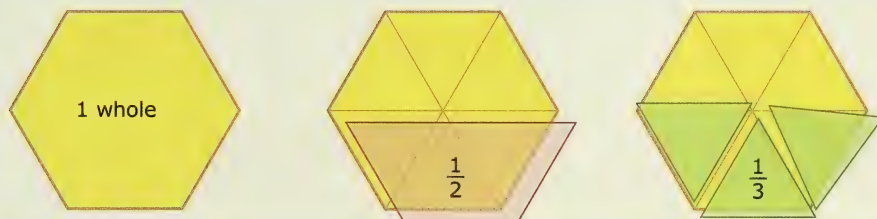
Lesson 2

SC 1.

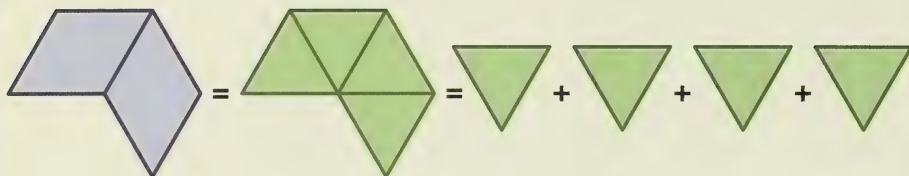
1. a. $\frac{1}{2} \div 3 = \frac{1}{6}$. Answers will vary. Example: $\frac{1}{2}$ of the rectangle is divided into three equal parts.

b. Answers will vary.

For example, you could use pattern blocks. With the hexagon representing one whole, the trapezoid would represent $\frac{1}{2}$. Since three triangles fit on the trapezoid and also on the rest of the area making up a hexagon, each triangle represents $\frac{1}{6}$. Separating a trapezoid into three equal parts gives you a green triangle.



2.a. Answers will vary. Example:

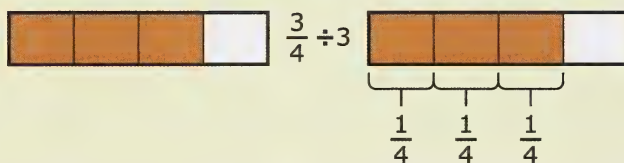


$$\frac{2}{3} \div 2 = \frac{1}{3}$$

Bruce McAskill et al., *MathLinks 8 Teacher's Resource* (Toronto: McGraw-Hill Ryerson, 2008), 273. Adapted by permission.

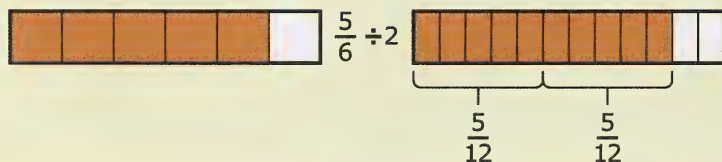
SC 2. Your answer may look different if you used another model to determine the quotient.

a. $\frac{1}{4}$



You may wonder why, in this model, the quarter sections did not have to be broken into subsections. That's because the shaded part is already split in three (right at the $\frac{1}{4}$ section marks themselves).

b. $\frac{5}{12}$

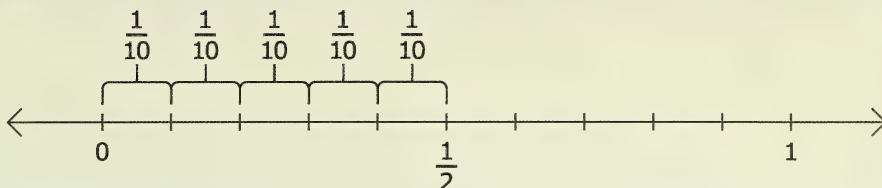


You can see that in this model, the one-sixth sections do have to be broken into subsections. By subdividing further into twelfths, you can see exactly which mark can be used to split the shaded part in two.

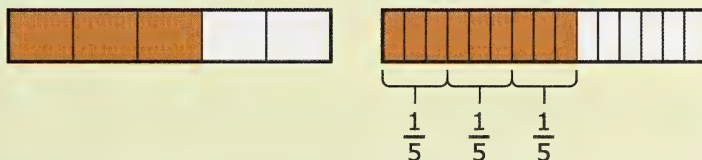
Bruce McAskill et al., *MathLinks 8 Teacher's Resource* (Toronto: McGraw-Hill Ryerson, 2008), 272. Reproduced by permission.

SC 3.

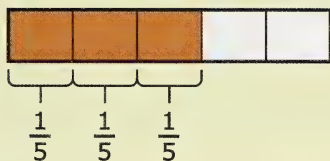
a. $\frac{1}{10}$



b. $\frac{1}{5}$

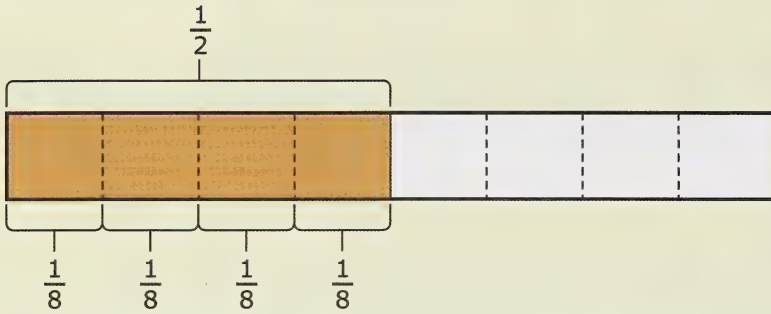


Notice that even without the subdivision of the fraction strip into one-fifths, you can see where the shaded region is split into three—right at the one-fifths marks. So, an alternate way to model this division is to place the brackets directly under the left strip.



Adapted from Bruce McAskill et al., *MathLinks 8 Teacher's Resource* (Toronto: McGraw-Hill Ryerson, 2008), BLM 13. Reproduced by permission.

SC 4.

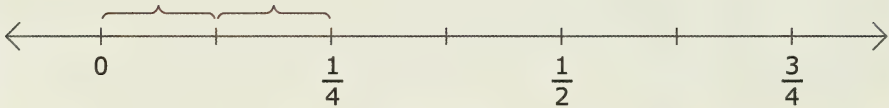


$$\frac{1}{2} \div 4 = \frac{1}{8}$$

Each student ate $\frac{1}{8}$ of a whole cake.

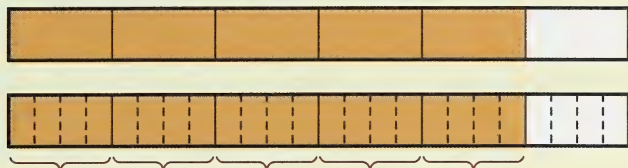
SC 5.

4. a. Draw and label a number line that show fourths. To model division by 2, cut each fourth into two equal parts.



There are eight parts in one whole, so each part is $\frac{1}{8}$. So, $\frac{1}{4} \div 2 = \frac{1}{8}$.

d. Use a diagram of a rectangle to represent $\frac{5}{6}$.



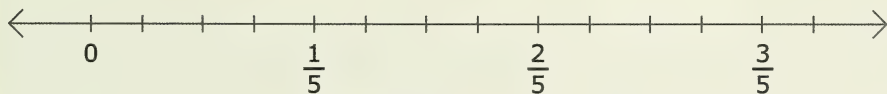
Divide each $\frac{1}{6}$ section of the rectangle into four equal parts. There is a total of 24 parts in the strip.

Each of the equal parts of $\frac{5}{6}$ is $\frac{5}{24}$. So, $\frac{5}{6} \div 4 = \frac{5}{24}$.

5. a. Draw and label a number line that shows fifths.

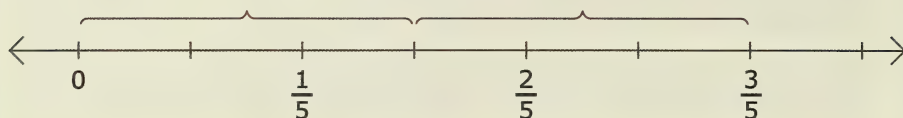


Cut each $\frac{1}{5}$ into two equal parts.



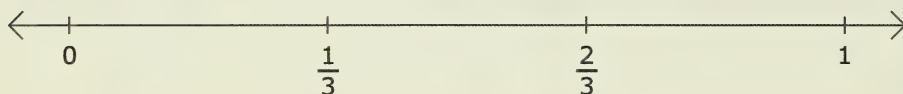
Operations with Fractions

Each part is $\frac{1}{10}$. Use brackets to cut $\frac{3}{5}$ into two equal parts.

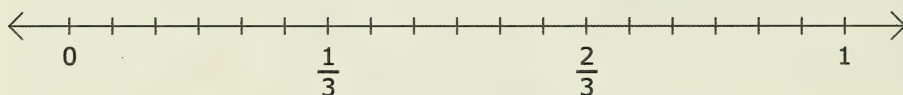


Each of the two parts is $\frac{3}{10}$. So, $\frac{3}{5} \div 2 = \frac{3}{10}$.

d. Draw and label a number line that shows thirds.

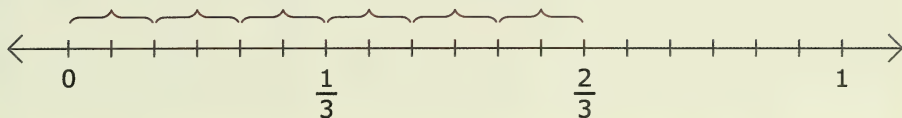


Cut each $\frac{1}{3}$ into six equal parts.



Note that cutting each $\frac{1}{3}$ into just three equal parts also puts enough marking in place to allow you to position the brackets properly.

Each part is $\frac{1}{18}$. Use brackets to cut $\frac{2}{3}$ into six equal parts.



Each of the six parts is $\frac{2}{18}$. So, $\frac{2}{3} \div 6 = \frac{2}{18} = \frac{1}{9}$.

Note that cutting each $\frac{1}{3}$ into just three parts also puts enough marking in place to allow you to position the brackets properly. Your final answer will be the same.

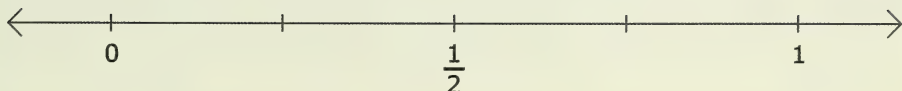
MathLinks 8 Solutions CD (Toronto: McGraw-Hill Ryerson, 2008), 209-211. Reproduced by permission.

SC 6.

6. a. To determine the fraction of a whole coconut in each serving, divide $\frac{1}{2}$ by 2.

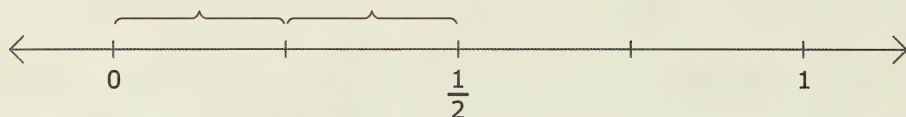
$$\frac{1}{2} \div 2 =$$

Draw and label a number line that shows halves.



Operations with Fractions

Cut each half into two equal parts.



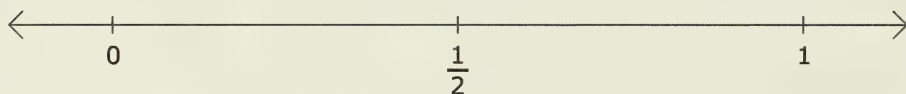
Each part is $\frac{1}{4}$. So, $\frac{1}{2} \div 2 = \frac{1}{4}$.

Each serving contains $\frac{1}{4}$ of a whole coconut.

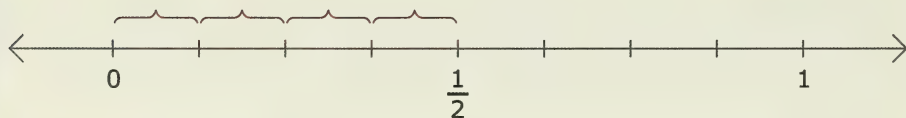
b. To determine the fraction of a coconut in each serving, divide $\frac{1}{2}$ by 4.

$$\frac{1}{2} \div 4 =$$

Draw and label a number line that shows halves.



Cut each half into four equal parts.



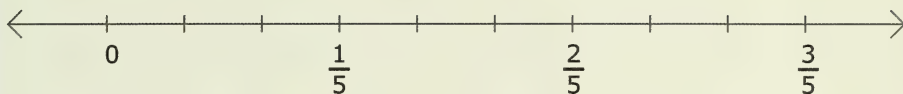
Each part is $\frac{1}{8}$. So, $\frac{1}{2} \div 4 = \frac{1}{8}$.

Each serving contains $\frac{1}{8}$ of a whole coconut.

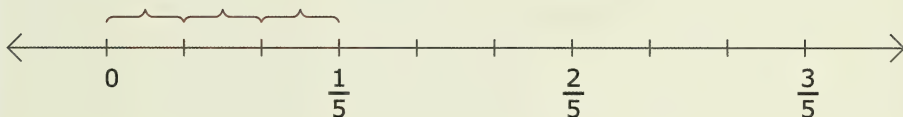
8. To find the fraction that represents the area of each of the provinces as a fraction of Canada, divide $\frac{1}{5}$ by 3.

$$\frac{1}{5} \div 3 =$$

Draw and label a number line that shows fifths.



Cut each $\frac{1}{5}$ into three equal parts.



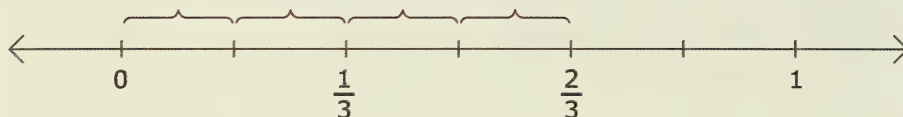
Each part is $\frac{1}{15}$. So, $\frac{1}{5} \div 3 = \frac{1}{15}$.

Each province is $\frac{1}{15}$ the area of Canada.

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Going Beyond

15. a.



Divide and label a number line in thirds. Divide each third into two equal parts.

Each part is $\frac{1}{6}$.

So, $\frac{2}{3}$ divided into four equal parts equals $\frac{1}{6}$.

b. Answers may vary. A sample answer follows:

The model shows that there are four sections in $\frac{2}{3}$ of length $\frac{1}{6}$.

That is the same as dividing $\frac{2}{3}$ by $\frac{1}{6}$.

So, $\frac{2}{3} \div \frac{1}{6} = 4$.

So yes, even with fractions you can switch the divisor and the quotient in a statement and still have it remain true.

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Lesson 3

SC 1.

1. Answers will vary. Example: The section of paper that is shaded both yellow and blue represents $\frac{1}{2}$ of $\frac{2}{3}$.

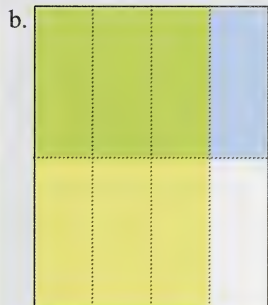
2. $\frac{2}{6}, \frac{1}{6}, \frac{3}{12}, \frac{6}{12}, \frac{9}{16}$

3. a. The numerators are multiplied to obtain the product.

b. The denominators are multiplied to obtain the product.

4. Answers will vary. Example: To find the product of two proper fractions, multiply the numerators and multiply the denominators.

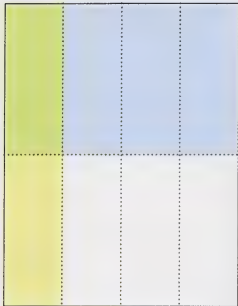
5. a. $\frac{3}{8}$



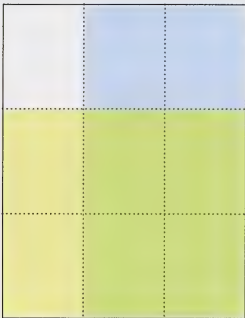
Bruce McAskill et al., *MathLinks 8 Teacher's Resource* (Toronto: McGraw-Hill Ryerson, 2008), 281-284. Reproduced by permission.

SC 2.

a. $\frac{1}{8}$

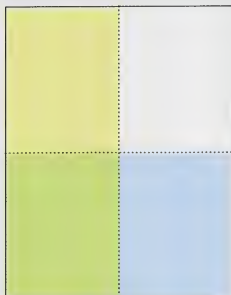


b. $\frac{4}{9}$

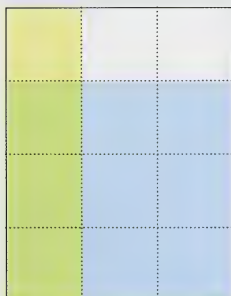


SC 3.

a. $\frac{1}{4}$



b. $\frac{3}{12}$



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SC 4.

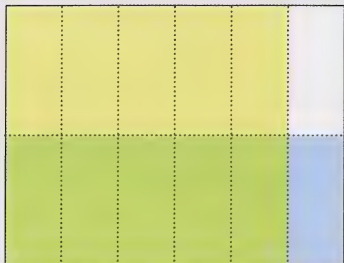
a. Estimates will vary. Example: 0; Answer: $\frac{2}{15}$

b. Estimates will vary. Example: 1; Answer: $\frac{2}{3}$

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SC 5.

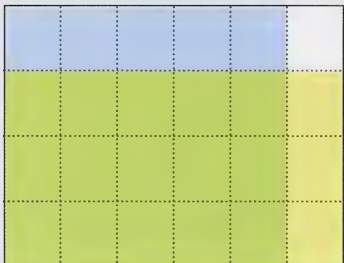
3. a. Fold a rectangular piece of paper into sixths along its length.



Open the paper, and then shade five-sixths of it yellow. Fold the paper in half across its width. Open the paper, and shade half of it blue. The folds make 12 equal rectangles. Five of them are shaded both yellow and blue, so they appear green.

$$\text{So, } \frac{5}{6} \times \frac{1}{2} = \frac{5}{12}.$$

b. Fold a rectangular piece of paper into fourths along its width.



Open the paper, and shade three-fourths of it yellow. Fold the paper into sixths along its length. Open the paper and shade five-sixths blue. The folds make 24 equal rectangles. Fifteen of them are shaded both yellow and blue, so they appear green.

$$\text{So, } \frac{3}{4} \times \frac{5}{6} = \frac{15}{24} \text{ or } \frac{5}{8}.$$

5. a. Estimates will vary. Example: $\frac{3}{8}$ is close to $\frac{1}{2}$, $\frac{2}{3}$ is close to $\frac{1}{2}$; $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$.

To multiply the fractions, multiply the numerators and multiply the denominators.

$$\frac{3}{8} \times \frac{2}{3} = \frac{6}{24} = \frac{1}{4}$$

The answer is the same as the estimate.

b. Estimates will vary. Example: $\frac{3}{7}$ is close to $\frac{1}{2}$, $\frac{1}{6}$ is close to 0: $\frac{1}{2} \times 0 = 0$.

To multiply the fractions, multiply the numerators and multiply the denominators.

$$\frac{3}{7} \times \frac{1}{6} = \frac{3}{42} = \frac{1}{14}$$

The answer is close to the estimate.

c. Estimates will vary. Example: $\frac{3}{4}$ is halfway between $\frac{1}{2}$ and 1: $1 \times \frac{1}{2} = \frac{1}{2}$.

To multiply the fractions, multiply the numerators and multiply the denominators.

$$\frac{3}{4} \times \frac{3}{4} = \frac{9}{16}$$

The answer is close to the estimate.

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SC 6.

7. To find the fraction of the pie that she ate, multiply $\frac{1}{2}$ by $\frac{1}{4}$.

$$\frac{1}{2} \times \frac{1}{4} = \frac{1}{8}$$

Tamar ate $\frac{1}{8}$ of a pie.

11. a. To find the fraction of gold medals, multiply $\frac{6}{7}$ by $\frac{7}{18}$.

$$\frac{6}{7} \times \frac{7}{18} = \frac{42}{126} = \frac{1}{3}$$

One-third of Canada's medals were gold medals.

b. To find the number of gold medals won, multiply $\frac{1}{3}$ by 84.

$$\frac{1}{3} \times 84 = \frac{84}{3} = 28$$

The paralympic athletes won 28 gold medals.

Operations with Fractions

12. Answers will vary.

Example: A bottle is $\frac{3}{4}$ full of juice. If Karen drinks $\frac{1}{2}$ of the juice in the bottle, what fraction of a full bottle did she drink?

Answer to this word problem:

Multiply $\frac{3}{4}$ by $\frac{1}{2}$.

$$\frac{3}{4} \times \frac{1}{2} = \frac{3}{8}$$

Karen drank $\frac{3}{8}$ of a bottle of juice.

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Going Beyond

15. a. To find the missing numerator, find what number multiplied by 1 equals 5: $5 \div 1 = 5$.

To find the missing denominator, find what number multiplied by 2 equals 16: $16 \div 2 = 8$.

The missing fraction is $\frac{5}{8}$.

b. Change the fraction $\frac{1}{3}$ to an equivalent form with a denominator that is a multiple of 7 and 3 (63): $\frac{1}{3} = \frac{21}{63}$.

To find the missing numerator, find what number multiplied by 3 equals 21: $21 \div 3 = 7$. To find the missing denominator, find what number multiplied by 7 equals 63: $63 \div 7 = 9$.

The missing fraction is $\frac{7}{9}$.

c. Change the fraction $\frac{1}{2}$ to an equivalent form with a denominator that is a multiple of 3 and 2 (12): $\frac{1}{2} = \frac{6}{12}$.

To find the missing numerator, find what number multiplied by 2 equals 6: $6 \div 2 = 3$. To find the missing denominator, find what number multiplied by 3 equals 12: $12 \div 3 = 4$.

The missing fraction is $\frac{3}{4}$.

d. Change the fraction $\frac{5}{8}$ to an equivalent form with a denominator that is a multiple of 8 and 4 (24): $\frac{5}{8} = \frac{15}{24}$.

To find the missing numerator, find what number multiplied by 3 equals 15: $15 \div 3 = 5$. To find the missing denominator, find what number multiplied by 4 equals 24: $24 \div 4 = 6$.

The missing fraction is $\frac{5}{6}$.

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Lesson 4

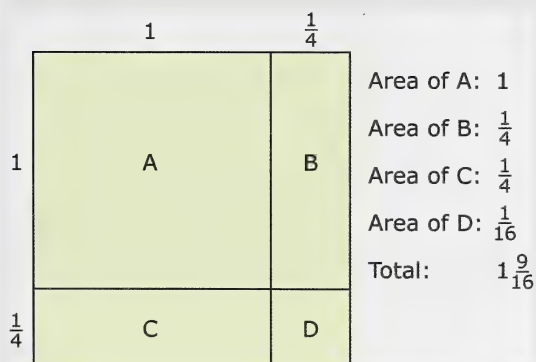
SC 1.

1. a. Methods may vary. Example: Multiply the length by the width in each section. A: 1; B: $\frac{1}{2}$; C: $\frac{1}{2}$; D: $\frac{1}{4}$. The total area is the sum of the areas of all sections. $2\frac{1}{4}$

b. $1\frac{1}{2} \times 1\frac{1}{2} = 2\frac{1}{4}$

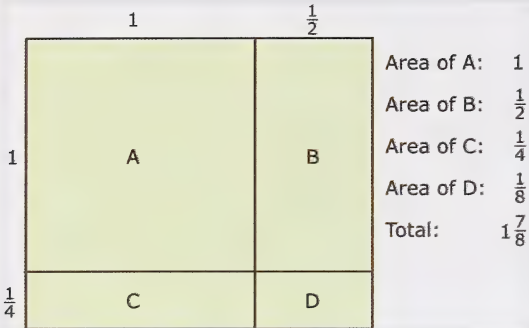
2. a. $6\frac{1}{4}$

b. $1\frac{9}{16}$



Operations with Fractions

c. $1\frac{7}{8}$



3.a.

| Multiplication of Mixed Numbers | Product Expressed as a Mixed Number | Multiplication of Improper Fractions | Product Expressed as an Improper Fraction |
|------------------------------------|-------------------------------------|--------------------------------------|---|
| $1\frac{1}{2} \times 1\frac{1}{2}$ | $2\frac{1}{4}$ | $\frac{3}{2} \times \frac{3}{2}$ | $\frac{9}{4}$ |
| $2\frac{1}{2} \times 2\frac{1}{2}$ | $6\frac{1}{4}$ | $\frac{5}{2} \times \frac{5}{2}$ | $\frac{25}{4}$ |
| $1\frac{1}{4} \times 1\frac{1}{4}$ | $1\frac{9}{16}$ | $\frac{5}{4} \times \frac{5}{4}$ | $\frac{25}{16}$ |
| $1\frac{1}{2} \times 1\frac{1}{4}$ | $1\frac{7}{8}$ | $\frac{3}{2} \times \frac{5}{4}$ | $\frac{15}{8}$ |

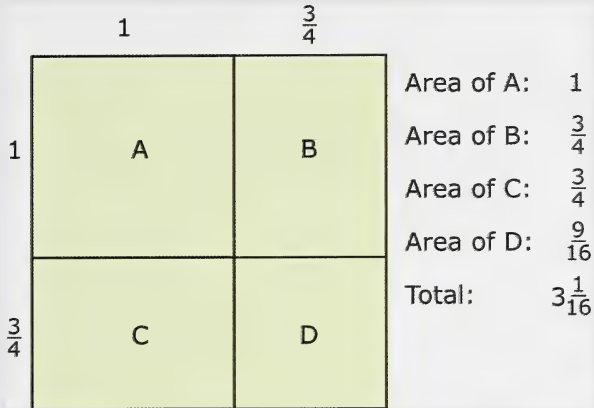
b. Answers will vary. Example: Multiply the numerators and multiply the denominators.

c. The rule is the same because to multiply two proper fractions, you multiply the numerators and multiply the denominators.

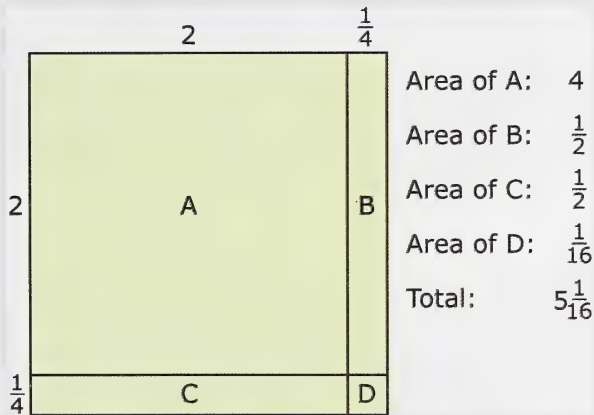
Bruce McAskill et al., *MathLinks 8 Teacher's Resource*(Toronto: McGraw-Hill Ryerson, 2008), 291. Reproduced by permission.

SC 2.

a. $3\frac{1}{16}$

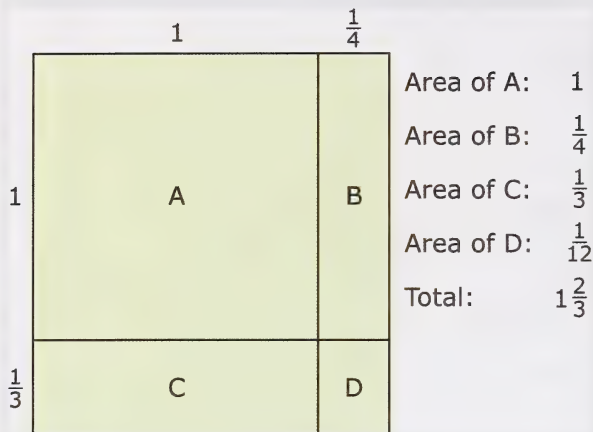


b. $5\frac{1}{16}$



Operations with Fractions

c. $1\frac{2}{3}$



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SC 3.

a. Estimates will vary. Example: 3; Answer: $3\frac{17}{20}$

b. Estimates will vary. Example: 4; Answer: $4\frac{7}{12}$

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SC 4.

4. a. In $\frac{11}{3}$, one whole is $\frac{3}{3}$; $\frac{11}{3} = \frac{3}{3} + \frac{3}{3} + \frac{3}{3} + \frac{2}{3}$. So, $\frac{11}{3} = 3\frac{2}{3}$.

c. In $\frac{25}{2}$, one whole is $\frac{2}{2}$;

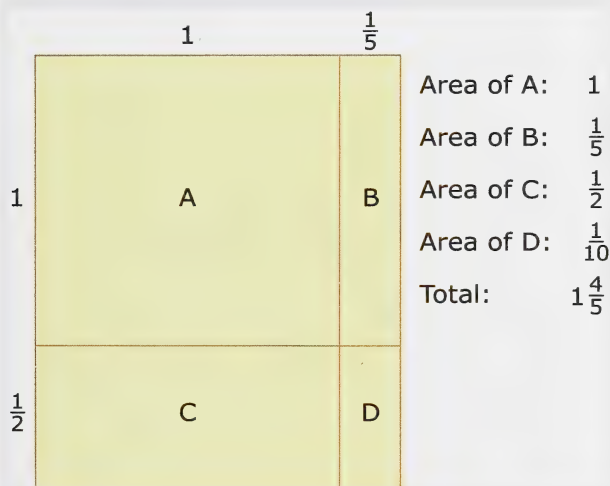
$$\frac{25}{2} = \frac{2}{2} + \frac{2}{2} + \frac{2}{2} + \frac{2}{2} + \frac{2}{2} + \frac{2}{2} + \frac{2}{2} + \frac{2}{2} + \frac{2}{2} + \frac{2}{2} + \frac{2}{2} + \frac{2}{2} + \frac{2}{2} + \frac{2}{2} + \frac{1}{2}. \text{ So, } \frac{25}{2} = 12\frac{1}{2}.$$

5. d. In $3\frac{4}{7}$, one whole is $\frac{7}{7}$; $3\frac{4}{7} = \frac{7}{7} + \frac{7}{7} + \frac{7}{7} + \frac{4}{7} = \frac{25}{7}$. So, $3\frac{4}{7} = \frac{25}{7}$.

6. d. The model represents the area of a rectangle with dimensions $2\frac{1}{2}$ by $2\frac{1}{4}$. The rectangle is divided into four regions. Section A has an area of 4. Section B has an area of $\frac{1}{2}$. Section C has an area of 1. Section D has an area of $\frac{1}{8}$. The sum of the four areas is $5\frac{5}{8}$. So,
 $2\frac{1}{2} \times 2\frac{1}{4} = 5\frac{5}{8}$.

7. c. The model represents the area of a rectangle with dimensions $1\frac{1}{2}$ by $2\frac{1}{3}$. The rectangle is divided into four regions. Section A has an area of 2. Section B has an area of 1. Section C has an area of $\frac{1}{3}$. Section D has an area of $\frac{1}{6}$. The sum of the four areas is $3\frac{1}{2}$. So,
 $1\frac{1}{2} \times 2\frac{1}{3} = 3\frac{1}{2}$.

d. The model represents the area of a rectangle with dimensions $1\frac{1}{5}$ by $1\frac{1}{2}$. The rectangle is divided into four regions.



Section A has an area of 1. Section B has an area of $\frac{1}{5}$. Section C has an area of $\frac{1}{2}$. Section D has an area of $\frac{1}{10}$. The sum of the four areas is $1\frac{4}{5}$. So, $1\frac{1}{5} \times 1\frac{1}{2} = 1\frac{4}{5}$.

8. Estimates may vary. Example: Estimate the product by multiplying the whole numbers closest to each mixed number.

b. $3\frac{3}{4}$ is close to 4: $5 \times 4 = 20$

To calculate $5 \times 3\frac{3}{4}$, write the mixed numbers as improper fractions: $5 \times 3\frac{3}{4} = \frac{5}{1} \times \frac{15}{4}$.

Multiply the numerators and multiply the denominators: $\frac{5}{1} \times \frac{15}{4} = \frac{75}{4} = 18\frac{3}{4}$. The answer is close to the estimate.

Operations with Fractions

c. $2\frac{1}{5}$ is closest to 2, $1\frac{2}{3}$ is closest to 2: $2 \times 2 = 4$

To calculate $2\frac{1}{5} \times 1\frac{2}{3}$, write the mixed numbers as improper fractions: $2\frac{1}{5} \times 1\frac{2}{3} = \frac{11}{5} \times \frac{5}{3}$.

Multiply the numerators and multiply the denominators: $\frac{11}{5} \times \frac{5}{3} = \frac{55}{15} = 3\frac{10}{15} = 3\frac{2}{3}$. The answer is close to the estimate.

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SC 5.

10. To determine the number of laps that equal 3 km, multiply $2\frac{1}{2}$ by 3:

$$2\frac{1}{2} \times 3 = \frac{5}{2} \times \frac{3}{1} = \frac{15}{2} = 7\frac{1}{2}.$$

There are $7\frac{1}{2}$ laps in 3 km.

14. To determine the cost of the living room carpet in relation to the den carpet, multiply $1\frac{3}{4}$ by $2\frac{1}{2}$: $1\frac{3}{4} \times 2\frac{1}{2} = \frac{7}{4} \times \frac{5}{2} = \frac{35}{8} = 4\frac{3}{8}$.

The cost of the living room carpet will be $4\frac{3}{8}$ times the cost of the den carpet.

17. Answers may vary. Example: The value of the product is smaller than the value of the mixed fraction.

Example: $\frac{1}{4} \times 2\frac{1}{2} = \frac{1}{4} \times \frac{5}{2} = \frac{5}{8}$

$$\frac{5}{8} < 2\frac{1}{2}$$

The value of the product is larger than the value of the proper fraction.

Example: $\frac{1}{4} \times 2\frac{1}{2} = \frac{1}{4} \times \frac{5}{2} = \frac{5}{8}$

$$\frac{5}{8} > \frac{1}{4}$$

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Going Beyond

19. a. If each fraction is changed to its improper fraction form, the numerator is 13 and denominator is twice the denominator of the previous term.

The next three terms are $\frac{13}{48}$, $\frac{13}{96}$, $\frac{13}{192}$.

b. Multiply each term by $\frac{3}{2}$ to get the next term.

The next three terms are $20\frac{1}{4}$, $30\frac{3}{8}$, $45\frac{9}{16}$.

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Lesson 5

SC 1.

1. a. Answer will vary. Example:



b. $3 \div \frac{1}{2} = 6$

2. a. Answer will vary. Example:



b. $2 \div \frac{1}{4} = 8$

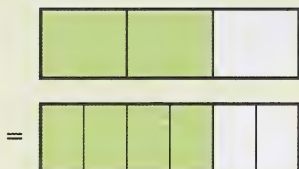
Operations with Fractions

3. a. Answer will vary. Example:



b. $\frac{3}{4} \div \frac{1}{4} = 3$

4. a. Answer will vary. Example:



b. $\frac{2}{3} \div \frac{1}{6} = 4$

5. a.

| Division |
|-------------------------------------|
| $\frac{3}{4} \div \frac{1}{4} = 3$ |
| $\frac{2}{3} \div \frac{1}{3} = 2$ |
| $\frac{8}{9} \div \frac{2}{9} = 4$ |
| $\frac{2}{3} \div \frac{1}{6} = 4$ |
| $\frac{1}{2} \div \frac{1}{12} = 6$ |
| $\frac{3}{4} \div \frac{3}{8} = 2$ |

b. Answers will vary. Example: The divisions are the same, except the fractions in the second column have common denominators.

c.

| Division With Equal Denominators |
|--|
| $\frac{3}{4} \div \frac{1}{4} = 3$ |
| $\frac{2}{3} \div \frac{1}{3} = 2$ |
| $\frac{8}{9} \div \frac{2}{9} = 4$ |
| $\frac{4}{6} \div \frac{1}{6} = 4$ |
| $\frac{6}{12} \div \frac{1}{12} = 6$ |
| $\frac{6}{8} \div \frac{3}{8} = 2$ |

d. A rule is to divide the numerators to get the numerator of the answer. The denominator of the answer is 1.

Operations with Fractions

6. a. See table in 5. a.

b.

| Multiplication |
|---------------------------------------|
| $\frac{3}{4} \times \frac{4}{1} = 3$ |
| $\frac{2}{3} \times \frac{3}{1} = 2$ |
| $\frac{8}{9} \times \frac{9}{2} = 4$ |
| $\frac{2}{3} \times \frac{6}{1} = 4$ |
| $\frac{1}{2} \times \frac{12}{1} = 6$ |
| $\frac{3}{4} \times \frac{3}{8} = 2$ |

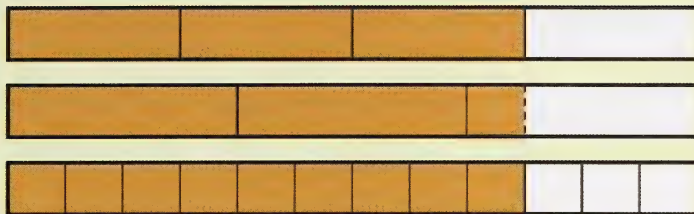
c. Answers will vary. Example: They are equivalent.

d. A rule is to switch the numerator and denominator of the fraction following the division sign, and then multiply.

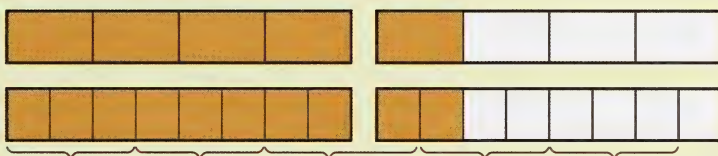
Bruce McAskill et al., *MathLinks 8 Teacher's Resource* (Toronto: McGraw-Hill Ryerson, 2008), 301. Reproduced by permission.

SC 2.

a. $2\frac{1}{4}$

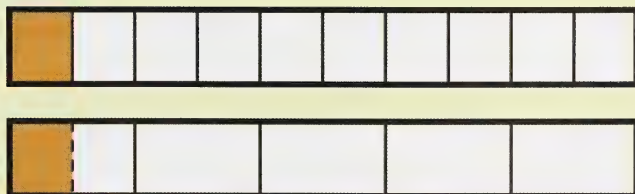


b. $3\frac{1}{3}$



$$\frac{5}{4} \div \frac{3}{8} = 3\frac{1}{3}$$

c. $\frac{1}{2}$



$$\frac{1}{10} \div \frac{1}{5} = \frac{1}{2}$$

Operations with Fractions

SC 3.

a. Estimates will vary. Possible estimate: 2

$$\begin{aligned}\text{Answer: } & \frac{4}{5} \div \frac{3}{10} \\ & \frac{4}{5} \times \frac{10}{3} \\ & \frac{40}{15} \\ & 2 \frac{10}{15} \\ & 2 \frac{2}{3}\end{aligned}$$

b. Estimates will vary. Possible estimates: $\frac{1}{3}$

$$\begin{aligned}\text{Answer: } & \frac{2}{9} \div \frac{5}{6} \\ & \frac{2}{9} \times \frac{6}{5} \\ & \frac{12}{45} \\ & \frac{4}{15}\end{aligned}$$

c. Estimates will vary. Possible estimate: $1 \frac{1}{2}$

$$\begin{aligned}\text{Answer: } & 3 \frac{1}{6} \div 1 \frac{2}{3} \\ & \frac{19}{6} \div \frac{5}{3} \\ & \frac{19}{6} \times \frac{3}{5} \\ & \frac{57}{30} \\ & 1 \frac{27}{30} \\ & 1 \frac{9}{10}\end{aligned}$$

SC 4.

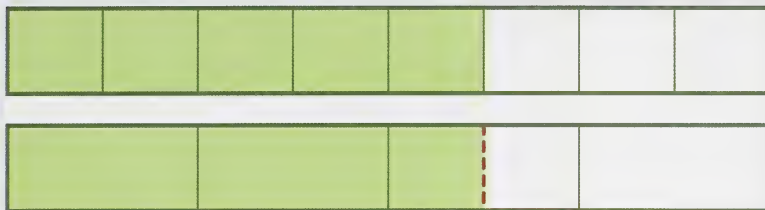
$$3 \div \frac{1}{6} = 3 \times \frac{6}{1}$$

18

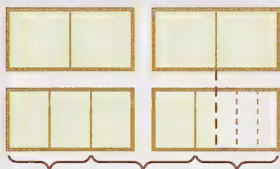
There are 18 servings in three trays of lasagna.

SC 5.

5. a. Divide a rectangle into eighths. Divide another rectangle into fourths. The diagram shows that the number of fourths in $\frac{5}{8}$ is $2\frac{1}{2}$. So, $\frac{5}{8} \div \frac{1}{4} = 2\frac{1}{2}$.



c. Divide two rectangles each in half to represent $1\frac{1}{2}$. Divide another two rectangles each into thirds. The diagram shows that the number of $\frac{2}{3}$ s in $1\frac{1}{2}$ is $2\frac{1}{4}$. So, $1\frac{1}{2} \div \frac{2}{3} = 2\frac{1}{4}$.



6. a. Divide a rectangle into tenths. Divide another rectangle into fifths. The diagram shows that the number of fifths in $\frac{9}{10}$ is $4\frac{1}{2}$. So, $\frac{9}{10} \div \frac{1}{5} = 4\frac{1}{2}$.



b. Divide a rectangle into fourths. Divide another rectangle into eighths. The diagram shows that the number of $\frac{3}{8}$ s in $\frac{1}{4}$ is $\frac{2}{3}$. So, $\frac{1}{4} \div \frac{3}{8} = \frac{2}{3}$.



Operations with Fractions

7. a. Write both fractions with a common denominator: $\frac{3}{5} \div \frac{9}{10} = \frac{6}{10} \div \frac{9}{10}$. Divide the numerators: $6 \div 9 = \frac{6}{9} = \frac{2}{3}$. So, $\frac{3}{5} \div \frac{9}{10} = \frac{2}{3}$.

c. Write both fractions with a common denominator: $3\frac{1}{3} \div 1\frac{5}{6} = \frac{20}{6} \div \frac{11}{6}$. Divide the numerators: $20 \div 11 = \frac{20}{11} = 1\frac{9}{11}$. So, $3\frac{1}{3} \div 1\frac{5}{6} = 1\frac{9}{11}$.

9. a. $\frac{3}{4} \div \frac{4}{5} = \frac{3}{4} \times \frac{5}{4} = \frac{15}{16}$

b. $1\frac{2}{3} \div 2\frac{5}{6} = \frac{5}{3} \div \frac{17}{6} = \frac{5}{3} \times \frac{6}{17} = \frac{30}{51} = \frac{10}{17}$

c. $12 \div \frac{3}{4} = \frac{12}{1} \times \frac{4}{3} = \frac{48}{3} = 16$

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SC 6.

11. To determine how many performers are in a 2-h show, divide 2 by $\frac{1}{4}$:

$2 \div \frac{1}{4} = \frac{2}{1} \times \frac{4}{1} = \frac{8}{1} = 8$. There are 8 performers in a 2-h show.

14. To determine the fraction of the energy used, divide 1 by $4\frac{1}{2}$: $1 \div 4\frac{1}{2} = \frac{1}{1} \div \frac{9}{2} = \frac{1}{1} \times \frac{2}{9} = \frac{2}{9}$.

The fluorescent light bulb will use $\frac{2}{9}$ of the energy used by an incandescent bulb.

Maybe you didn't see why you divide 1 by $4\frac{1}{2}$. Sometimes it helps to think of a similar problem simplified so fractions and mixed numbers are approximated by whole numbers.

Suppose the information was that an incandescent bulb uses 4 times the energy as a fluorescent. What fraction of the energy used by the incandescent bulb does the fluorescent bulb use? You may reason that the answer is $\frac{1}{4}$ and that it is the reciprocal of 4 that gives you that answer.

You see reciprocal coming up when you change the order of the comparison. Think of this statement, "Bon can run 2 times as fast as Bill." It follows that Bill can run only $\frac{1}{2}$ as fast as Bon. The fraction $\frac{1}{2}$ and the whole number 2 are reciprocals, aren't they?

In order to find the reciprocal of $4\frac{1}{2}$, you divide 1 by this mixed number. That's how you can arrive at the answer to question 14.

16. To determine how many times bigger Asia is than South America, divide $\frac{3}{10}$ by $\frac{3}{25}$:

$$\frac{3}{10} \div \frac{3}{25} = \frac{3}{10} \times \frac{25}{3}$$

$$= \frac{75}{30}$$

$$= 2 \frac{15}{30}$$

$$= 2 \frac{1}{2}$$

Asia is $2 \frac{1}{2}$ times as big as South America.

Do you have trouble figuring out what to divide by what in question 16? This kind of question is asking for a factor that tells how two quantities compare. In this kind of question you may see wording like “how many times more?” or “how many times as big?”

To guide your interpretation of the words of problem 16, make up an easier problem but one that is still the same kind. The following is an example:

Yoshimi had 10 apples and Jim had 2 apples. How many more times more apples does Yoshimi have?

You would take the larger number and divide by the smaller number: $10 \div 2 = 5$. Now 5 is the factor used for comparison. You conclude that Yoshimi has 5 times as many apples as Jim.

So the same thing applies in question 16. Take the larger area (Asia) and divide by the smaller area (South America) to get the answer.

18. a. The answer is no. How you explain this answer may vary.

Example: The reciprocal of $\frac{5}{6}$ is $\frac{6}{5}$.

Example: A proper fraction has a denominator (bottom) that is larger than the numerator (top). As soon as you take the reciprocal (flip it) then automatically the numerator becomes larger so the reciprocal is an improper fraction.

b. The answer is no, but how you explain this answer may vary.

Example: $\frac{9}{10} \times \frac{5}{6} = \frac{45}{60}$ or $\frac{3}{4}$.

If you multiply using proper fractions (e.g., the denominator is larger than the numerator), then the denominator will be larger than the numerator in the answer. So your answer will never be greater than 1.

c. The answer is yes, but how you explain this answer may vary.

$$\text{Example: } \frac{9}{10} \div \frac{5}{6} = \frac{9}{10} \times \frac{6}{5} = \frac{54}{50} = 1 \frac{4}{50} = 1 \frac{2}{25}.$$

20. To determine the fraction of Earth that Canada covers, divide $\frac{1}{30}$ by $1\frac{2}{3}$:

$$\begin{aligned} \frac{1}{30} \div 1\frac{2}{3} \\ &= \frac{1}{30} \div \frac{5}{3} \\ &= \frac{1}{30} \times \frac{3}{5} \\ &= \frac{3}{150} = \frac{1}{50} \end{aligned}$$

Canada covers $\frac{1}{50}$ of Earth's surface.

Are you clear on this solution? If not, you may want to think about a similar problem involving Jim and Yoshimi from the solution to textbook question 16:

Say that Yoshimi had 10 apples and you know she has 5 times as many apples as Jim. Then how many apples does Jim have?

$$10 \div 5 = 2$$

Jim must have 2 apples.

The same idea applies with question 20. If Russia is $\frac{1}{30}$ of Earth's surface and it is $1\frac{2}{3}$ times greater in area than Canada, you can divide these numbers to find the area of Earth that Canada covers.

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Lesson 6

SC 1.

$$\begin{aligned} \frac{2}{9} \times 190\,000 &= \frac{2}{9} \times \frac{190\,000}{1} && \text{(according to a rule for multiplication of fractions)} \\ &= \frac{2 \times 190\,000}{9 \times 1} \\ &= \frac{380\,000}{9} && \text{(on your calculator)} \\ &= 380\,000 \div 9 \\ &= 42\,000 \end{aligned}$$

SC 2.

Number of Aboriginal people living off reserves:

$$190\,000 - 42\,000 = 148\,000$$

This is based on approximate numbers,
so the answer is an approximate value.

Number of Aboriginal people living in urban areas:

$$148\,000 \times \frac{4}{5} = \frac{148\,000}{1} \times \frac{4}{5}$$

$$= \frac{148\,000 \times 4}{1 \times 5}$$

According to a rule for multiplication of fractions,
multiply numerators together, and then multiply
the denominators together.

$$= 118\,000$$

Approximate.

The number of Aboriginal people living in urban areas is approximately 118 000.

SC 3.

a.

$$\frac{4}{5} \times \left[190\,000 - \left(\frac{2}{9} \times 190\,000 \right) \right]$$

population on reserve

population off reserve

urban population

Operations with Fractions

b. $\frac{4}{5} \times \left[190\,000 - \left(\frac{2}{9} \times 190\,000 \right) \right]$

$$\approx \frac{4}{5} \times (190\,000 - 42\,222)$$

Evaluate the expression in round brackets and round off the results for ease of calculation; change brackets to round ones.

$$= \frac{4}{5} \times 147\,778$$

Evaluate the expression in remaining brackets and remove brackets.

$$= \frac{4}{5} \times \frac{147\,778}{1}$$

Express the whole number as a fraction.

$$= \frac{4 \times 147\,778}{5 \times 1}$$

Apply rule for multiplication of fractions.

$$= \frac{591\,112}{5}$$

$$= 591\,110 \div 5$$

$$\approx 118\,223$$

Use calculator to divide; then round off.

The mathematical expression for the population of Aboriginal people living in urban areas is approximately 118 224 (or about 118 000 to the nearest thousand). This answer is close to the answer for SC 2. So you could conclude that the mathematical expression is correct.

Notice that writing and evaluating a mathematical expression is another way to solve some mathematical problems. You may prefer this method to solving a mathematical problem in separate steps.

SC 4.

a. $\frac{1}{6}$

b. $2\frac{2}{3}$

c. $1\frac{15}{16}$

SC 5.

Do the calculation in stages.

Amount earned at regular rate:

$$35 \times 15 = 525$$

The amount earned at regular rate is \$525.

Number hours worked beyond the regular hours in the week:

$$41 - 35 = 6$$

The number of hours extra is 6 h.

Number of hours paid for the extra hours worked:

$$\begin{aligned} 6 \times 1\frac{1}{3} &= 6 \times \frac{4}{3} \\ &= 8 \end{aligned}$$

The number of hours paid for the extra hours worked is 8 h.

Amount earned for extra hours worked:

$$8 \times 15 = 120$$

The amount that Ron earned for extra hours is \$120.

Total earned:

$$525 + 120 = 645$$

Ron earned \$645 for working 41 h in one week.

SC 6.

$$\begin{aligned} 4. \text{ a. } \frac{3}{4} - \frac{1}{2} \times \frac{2}{3} & \quad \text{Multiply.} \\ = \frac{3}{4} - \frac{2}{6} & \quad \text{Subtract.} \\ = \frac{9}{12} - \frac{4}{12} \\ = \frac{5}{12} \end{aligned}$$

Operations with Fractions

b. $2\frac{1}{5} \div \left(\frac{4}{5} - \frac{1}{4}\right)$ Brackets.

$$= 2\frac{1}{5} \div \left(\frac{16}{20} - \frac{5}{20}\right)$$

$$= 2\frac{1}{5} \div \frac{11}{20} \quad \text{Divide.}$$

$$= \frac{11}{5} \div \frac{11}{20}$$

$$= \frac{11}{5} \times \frac{20}{11}$$

$$= \frac{220}{55}$$

$$= 4$$

c. $3\frac{1}{2} + 2\frac{1}{2} \times \left(1\frac{1}{4} - \frac{3}{4}\right)$ Brackets.

$$= 3\frac{1}{2} + 2\frac{1}{2} \times \left(\frac{5}{4} - \frac{3}{4}\right)$$

$$= 3\frac{1}{2} + 2\frac{1}{2} \times \frac{2}{4} \quad \text{Multiply.}$$

$$= 3\frac{1}{2} + \frac{5}{2} \times \frac{2}{4}$$

$$= 3\frac{1}{2} + \frac{10}{8} \quad \text{Add.}$$

$$= \frac{7}{2} + \frac{10}{8}$$

$$= \frac{14}{4} + \frac{5}{4}$$

$$= \frac{19}{4}$$

$$= 4\frac{3}{4}$$

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SC 7.

6. To determine the amount earned for each number of hours, multiply \$16 by 35. Then subtract 35 from the number of hours worked. Multiply this difference by \$24:

$$\$16 \times 1\frac{1}{2} = 16 \times \frac{3}{2} = \frac{48}{2} = 24$$

a. $16 \times 35 + (36 - 35) \times 24$ Brackets.

$$= 16 \times 35 + 1 \times 24 \quad \text{Multiply.}$$

$$= 560 + 24 \quad \text{Add.}$$

$$= 584$$

He earns \$584 for working 36 h.

$$\begin{aligned}
 \text{b. } & 16 \times 35 + (39 - 35) \times 24 && \text{Brackets.} \\
 & = 16 \times 35 + 4 \times 24 && \text{Multiply.} \\
 & = 560 + 96 && \text{Add.} \\
 & = 656
 \end{aligned}$$

He earns \$656 for working 39 h.

$$\begin{aligned}
 \text{c. } & 16 \times 35 + (42 - 35) \times 24 && \text{Brackets.} \\
 & = 16 \times 35 + 7 \times 24 && \text{Multiply.} \\
 & = 560 + 168 && \text{Add.} \\
 & = 728
 \end{aligned}$$

He earns \$728 for working 42 h.

$$\begin{aligned}
 \text{d. } & 16 \times 35 + \left(37\frac{1}{2} - 35\right) \times 24 && \text{Brackets.} \\
 & = 16 \times 35 + 2\frac{1}{2} \times 24 && \text{Multiply.} \\
 & = 560 + \frac{5}{2} \times 24 && \text{Add.} \\
 & = 560 + 60 \\
 & = 620
 \end{aligned}$$

He earns \$620 for working $37\frac{1}{2}$ h.

11. a. To determine the total number of pages sold, multiply 3 by $\frac{1}{2}$, multiply 5 by $\frac{1}{4}$, and multiply 12 by $\frac{1}{8}$. Find the sum of the three products:

$$\begin{aligned}
 & 3 \times \frac{1}{2} + 5 \times \frac{1}{4} + 12 \times \frac{1}{8} && \text{Multiply.} \\
 & = \frac{3}{2} + \frac{5}{4} + \frac{12}{8} && \text{Add.} \\
 & = \frac{12}{8} + \frac{10}{8} + \frac{12}{8} \\
 & = \frac{34}{8} \\
 & = 4\frac{1}{4}
 \end{aligned}$$

A total of $4\frac{1}{4}$ pages of advertising were sold.

| Size of Advertising | Price | Number Sold |
|---------------------|-------|-------------|
| $\frac{1}{2}$ page | \$110 | 3 |
| $\frac{1}{4}$ page | \$60 | 5 |
| $\frac{1}{8}$ page | \$35 | 12 |

b. To determine the total revenue, add the products $\$110 \times 3$, $\$60 \times 5$, and $\$35 \times 12$:

$$\begin{aligned}
 &\$110 \times 3 + \$60 \times 5 + \$35 \times 12 && \text{Multiply.} \\
 &= \$330 + \$300 + \$420 && \text{Add.} \\
 &= \$1050
 \end{aligned}$$

The total revenue from advertising was \$1050.

c. To determine the average revenue per page, divide \$1050 by $4\frac{1}{4}$:

$$\begin{aligned}
 &\$1050 \div 4\frac{1}{4} \\
 &= 1050 \div \frac{17}{4} \\
 &= 1050 \times \frac{4}{17} \\
 &= \frac{4200}{17} \\
 &= 247.06
 \end{aligned}$$

The average revenue per page of advertising is about \$247.06.

12. To determine Marjorie's allowance, calculate the following:

$$\begin{aligned}
 &5 \div \left[1 - \left(\frac{1}{2} + \frac{1}{4} + \frac{1}{8} \right) \right] && \text{Brackets.} \\
 &= 5 \div \left[1 - \left(\frac{4}{8} + \frac{2}{8} + \frac{1}{8} \right) \right] \\
 &5 \div \left[1 - \frac{7}{8} \right] && \text{Brackets.} \\
 &5 \div \frac{1}{8} && \text{Divide.} \\
 &= 5 \times 8 \\
 &= 40
 \end{aligned}$$

Marjorie's allowance was \$40.

13. a. $\frac{5}{2} \times \left(\frac{3}{5} - \frac{2}{5} \right) + \frac{1}{2} = 1$

$\frac{5}{2} \times \left(\frac{3}{5} - \frac{2}{5} + \frac{1}{2} \right)$ Brackets.

$= \frac{5}{2} \times \frac{1}{5} + \frac{1}{2}$ Multiply.

$= \frac{5}{10} + \frac{1}{2}$ Add.

$= \frac{5}{10} + \frac{5}{10}$

$= \frac{10}{10}$ or 1

b. $1\frac{1}{2} + 2\frac{1}{2} \div \left(\frac{3}{4} - \frac{1}{8} \right) = 5\frac{1}{2}$

$1\frac{1}{2} + 2\frac{1}{2} \div \left(\frac{3}{4} - \frac{1}{8} \right)$ Brackets.

$= 1\frac{1}{2} + 2\frac{1}{2} \div \left(\frac{6}{8} - \frac{1}{8} \right)$

$= 1\frac{1}{2} + 2\frac{1}{2} \div \frac{5}{8}$ Divide.

$= 1\frac{1}{2} + \frac{5}{2} \times \frac{8}{5}$

$= \frac{15}{10} + \frac{40}{10}$ Add.

$= \frac{55}{10}$ or $5\frac{1}{2}$

c. $\left(\frac{2}{3} - \frac{1}{6} + \frac{5}{6} \right) \div \frac{16}{9} = \frac{3}{4}$

$\left(\frac{2}{3} - \frac{1}{6} + \frac{5}{6} \right) \div \frac{16}{9}$ Brackets.

$= \left(\frac{4}{6} - \frac{1}{6} + \frac{5}{6} \right) \div \frac{16}{9}$

$= \frac{8}{6} \div \frac{16}{9}$ Divide.

$= \frac{8}{6} \times \frac{9}{16}$

$= \frac{72}{96}$ or $\frac{3}{4}$

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Going Beyond

16. To determine the number of black notes, add 1 to $1\frac{4}{9}$, and then divide 88 by this sum:

$$\begin{aligned}
 &\text{There are 36 black notes.} && 88 \div \left(1 + 1\frac{4}{9}\right) && \text{Brackets.} \\
 & && = 88 \div \left(\frac{9}{9} + \frac{13}{9}\right) && \\
 &\text{To find the number of white notes, subtract the} && = 88 \div \frac{22}{9} && \text{Divide.} \\
 &\text{number of black notes from 88:} && = 88 \times \frac{9}{22} && \\
 &88 - 36 = 52 && = \frac{792}{22} \text{ or } 36 &&
 \end{aligned}$$

There are 52 white notes.

17. First find the number of CDs in the large rack by adding $1 + \frac{1}{2} + \frac{1}{4}$ and dividing 224 by this sum:

$$\begin{aligned}
 &\text{There are 128 CDs in the large rack.} && 224 \div \left(1 + \frac{1}{2} + \frac{1}{4}\right) && \text{Brackets.} \\
 & && = 224 \div \left(\frac{4}{4} + \frac{2}{4} + \frac{1}{4}\right) && \\
 &\text{There are half as many in the medium rack:} && = 224 \div \frac{7}{4} && \text{Divide.} \\
 &128 \div 2 = 64 && = 224 \times \frac{4}{7} && \\
 & && = \frac{896}{7} && \\
 &\text{There are half as many in the small rack:} && = 128 && \\
 &64 \div 2 = 32 && &&
 \end{aligned}$$

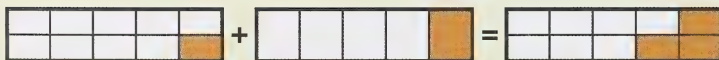
The large, medium, and small racks hold 128, 64, and 32 CDs, respectively.

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Unit 3 Summary

SC 1.

1. a.



$$\frac{3}{10}$$

The sum represents the portion of Canada's coastline falling within these ecozones.

b. $\frac{4}{21}$

$$2. 1 - \left(\frac{1}{3} + \frac{1}{2}\right) = \frac{1}{6}; 1 - \frac{1}{3} = \frac{2}{3} \text{ and } \frac{2}{3} - \frac{1}{2} = \frac{1}{6}$$

Bruce McAskill et al., *MathLinks 8 Teacher's Resource* (Toronto: McGraw-Hill Ryerson, 2008), 290. Reproduced by permission.

SC 2.

Turn to page 495 of the Answers section of your textbook. Check your answers with those provided.

Still not clear about some of the answers to the Review questions? Then e-mail or talk to your teacher to discuss any concepts you may still be struggling with.

SC 3.

1. D

$$\begin{aligned} 4 \times \frac{1}{3} &= \frac{4}{3} \\ &= 1\frac{1}{3} \end{aligned}$$

2. C

$$\frac{4}{5} \div \frac{2}{3} = \frac{4}{5} \times \frac{3}{2}$$

3. B

The reciprocal of $\frac{2}{3}$ is $\frac{3}{2}$.

$$\begin{aligned} 1 \div \frac{2}{3} &= 1 \times \frac{3}{2} \\ &= \frac{3}{2} \end{aligned}$$

Operations with Fractions

4. C

$$\begin{aligned}\frac{1}{2} \times \left(\frac{4}{3} - \frac{1}{6} \right) + \frac{3}{4} &= \frac{1}{2} \times \left(\frac{8}{6} - \frac{1}{6} \right) + \frac{3}{4} && \text{Brackets.} \\ &= \frac{1}{2} \times \frac{7}{6} + \frac{3}{4} && \text{Multiply.} \\ &= \frac{7}{12} + \frac{3}{4} && \text{Add.} \\ &= \frac{7}{12} + \frac{9}{12} \\ &= \frac{16}{12} \\ &= 1\frac{1}{3}\end{aligned}$$

5. A

$$\begin{aligned}\frac{3}{4} \div \frac{5}{12} &= \frac{3}{4} \times \frac{12}{5} \\ &= \frac{36}{20} \\ &= \frac{9}{5}\end{aligned}$$

6. The product of a fraction and its reciprocal is 1. Example:

$$\begin{aligned}\frac{3}{4} \times \frac{4}{3} &= \frac{12}{12} \\ &= 1\end{aligned}$$

$$\begin{aligned}7. \quad 2\frac{2}{3} \div 4\frac{2}{3} &= \frac{8}{3} \div \frac{14}{3} \\ &= \frac{8}{3} \times \frac{3}{14} \\ &= \frac{24}{42} \\ &= \frac{4}{7}\end{aligned}$$

$$\begin{aligned}8. \quad 2\frac{1}{4} \times 1\frac{1}{3} &= \frac{9}{4} \times \frac{4}{3} \\ &= \frac{36}{12} \\ &= 3\end{aligned}$$

$$\begin{aligned}9. \text{ a. } \frac{3}{8} \times \frac{5}{6} &= \frac{15}{48} \\ &= \frac{5}{16}\end{aligned}$$

$$\text{b. } \frac{6}{5} \div \frac{7}{10} = \frac{6}{5} \times \frac{10}{7}$$

$$= \frac{60}{35} \quad \text{Reduce by dividing the numerator (top) and denominator (bottom) by 5.}$$

$$= \frac{12}{7} \quad \text{Provide the final answer in lowest terms by changing the improper fraction into a mixed fraction.}$$

$$= 1\frac{5}{7}$$

$$\text{c. } 3\frac{3}{5} \times \frac{3}{8} = \frac{18}{5} \times \frac{3}{8}$$

$$= \frac{54}{40}$$

$$= \frac{27}{20}$$

$$= 1\frac{7}{20}$$

$$\text{d. } \frac{9}{10} \div 2\frac{1}{2} = \frac{9}{10} \div \frac{5}{2}$$

$$= \frac{9}{10} \times \frac{2}{5}$$

$$= \frac{18}{50}$$

$$= \frac{9}{25}$$

$$\text{e. } \left(1\frac{1}{4} + \frac{3}{4}\right) \div 1\frac{1}{2} - 1\frac{1}{3} = \left(\frac{5}{4} + \frac{3}{4}\right) \div 1\frac{1}{2} - 1\frac{1}{3} \quad \text{Brackets.}$$

$$= \frac{8}{4} \div 1\frac{1}{2} - 1\frac{1}{3} \quad \text{Divide.}$$

$$= \frac{8}{4} \div \frac{3}{2} - 1\frac{1}{3}$$

$$= \frac{8}{4} \times \frac{2}{3} - 1\frac{1}{3}$$

$$= \frac{16}{12} - 1\frac{1}{3}$$

$$= \frac{16}{12} - \frac{4}{3} \quad \text{Subtract.}$$

$$= \frac{16}{12} - \frac{16}{12}$$

$$= 0$$

10. To determine how much Leisha earned, multiply \$14 by $6\frac{1}{2}$:

$$14 \times 6\frac{1}{2} = 14 \times \frac{13}{2}$$

$$= \frac{182}{2}$$

$$= 91$$

Leisha earned \$91.

11. a. To determine what fraction of a box Chad eats per day, divide $\frac{3}{4}$ by 7:

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$$\begin{aligned}\frac{3}{4} \div \frac{7}{1} &= \frac{3}{4} \times \frac{1}{7} \\ &= \frac{3}{28}\end{aligned}$$

Chad eats $\frac{3}{28}$ of a box of granola per day.

b. To determine how many boxes of granola Chad eats per year, multiply 365 by $\frac{3}{28}$:

$$\begin{aligned}365 \times \frac{3}{28} &= \frac{1095}{28} \\ &= 39 \frac{3}{28}\end{aligned}$$

Chad eats approximately 39 boxes of granola a year.

12. To determine how many bits equal 16 bytes, divide 16 by $\frac{1}{8}$:

$$\begin{aligned}16 \div \frac{1}{8} &= 16 \times 8 \\ &= 128\end{aligned}$$

There are 128 bits in 16 bytes.

13. To determine how many sheets are used, multiply 500 by $1\frac{3}{4}$:

$$\begin{aligned}500 \times 1\frac{3}{4} &= 500 \times \frac{7}{4} \\ &= \frac{3500}{4} \\ &= 875\end{aligned}$$

The number of sheets used is 875.

14. To determine how long it will take Lianne to save enough money for the DVD player, subtract $\frac{3}{4}$ from 1, and then divide $2\frac{1}{2}$ by this difference:

$$\begin{aligned}2\frac{1}{2} \div \left(1 - \frac{3}{4}\right) &= 2\frac{1}{2} \div \left(\frac{4}{4} - \frac{3}{4}\right) && \text{Brackets.} \\ &= 2\frac{1}{2} \div \frac{1}{4} && \text{Divide.} \\ &= \frac{5}{2} \times \frac{4}{1} \\ &= \frac{20}{2} \text{ or } 10\end{aligned}$$

It will take Lianne 10 weeks to save enough money for the DVD player.

15. a. To determine how many carousels turn counterclockwise, multiply 100 by $\frac{9}{20}$:

$$100 \times \frac{9}{20} = \frac{900}{20} = 45$$

Forty-five carousels out of 100 turn counterclockwise.

b. To determine how many carousels turn either way, do the following computation:

$$\begin{aligned} 100 \times \left[1 - \left(\frac{9}{20} + \frac{3}{10} \right) \right] &= 100 \times \left[1 - \left(\frac{9}{20} + \frac{6}{20} \right) \right] && \text{Brackets.} \\ &= 100 \times \left[1 - \frac{15}{20} \right] && \text{Brackets.} \\ &= 100 \times \left[\frac{20}{20} - \frac{15}{20} \right] \\ &= 100 \times \frac{5}{20} && \text{Multiply.} \\ &= \frac{500}{20} \text{ or } 25 \end{aligned}$$

Twenty-five out of 100 carousels turn either way.

c. To get a better handle on the problem, you may want to use abbreviations. Call the carousels that always turn clockwise as CW-C and those that always turn counterclockwise as CCW-C. Then the problem can be stated this way:

How many times the number of CW-Cs is the number of CCW-Cs?

So you have to determine how many times the number of CCW-Cs is of the number of CW-Cs. To do that, you have to divide $\frac{9}{20}$ by $\frac{3}{10}$:

$$\begin{aligned} &\frac{9}{20} \div \frac{3}{10} \\ &= \frac{9}{20} \times \frac{10}{3} \\ &= \frac{90}{60} \\ &= 1 \frac{30}{60} \text{ or } 1 \frac{1}{2} \end{aligned}$$

The number of CCW-Cs is $1 \frac{1}{2}$ times the number of CW-Cs.

Paraphrasing this statement into the language used in the problem, the concluding statement is as follows:

The number of carousels that always turn counterclockwise is $1 \frac{1}{2}$ times the number of carousels that always turn clockwise.

d. To determine the number of carousels that were included in the survey, divide 75 by $\frac{3}{10}$:

$$\begin{aligned}75 &\div \frac{3}{10} \\&= 75 \times \frac{10}{3} \\&= \frac{750}{3} \\&= 250\end{aligned}$$

There were 250 carousels included in the survey.

Adapted from *MathLinks 8 Solutions CD* (Toronto: McGraw-Hill Ryerson, 2008), 238. Reproduced by permission.

